Power Semiconductor Diodes

A power semiconductor diode acts as a *switch*. It can be assumed as an ideal switch for most applications, but in practice this is not the case. It is similar to an ordinary *pn*-junction signal diode, but has larger power, voltage and current handling capabilities. The frequency response (switching speed) is low compared to signal diodes.

Diode Characteristics

A power diode is a two-terminal *pn*-junction device, as shown in Figures (1a,b).

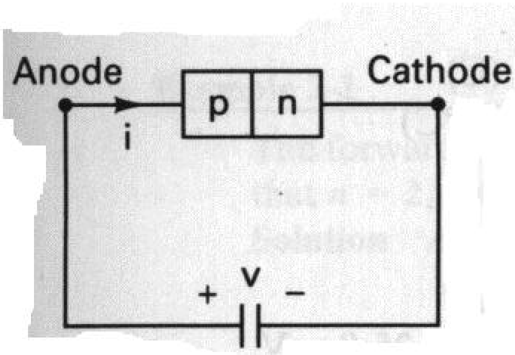

Figure (1a): pn-junction

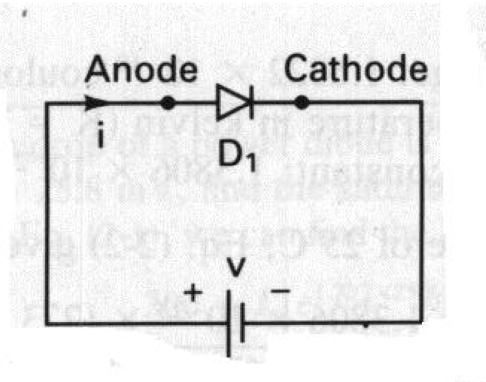

Figure (1b): Diode Symbol

When the *anode* potential is positive with respect to the *cathode*, the diode is said to **be forward biased** and the diode **conducts**. A conducting diode has a relatively small forward voltage drop across it. When the *cathode* potential is positive with respect to the anode, the *diode* is said to be **reverse biased**. Under reverse-biased conditions, a small reverse current (known as the *leakage current*) in the range of microampere to milliampere flows. The leakage current increases slowly in magnitude with the reverse voltage until the *avalanche* or *zener voltage* reached.

Figure (2a) shows the steady-state *v-i* characteristics of a diode. However, for most applications, a diode can be regarded as an ideal switch with the characteristics shown in Figure (2b).

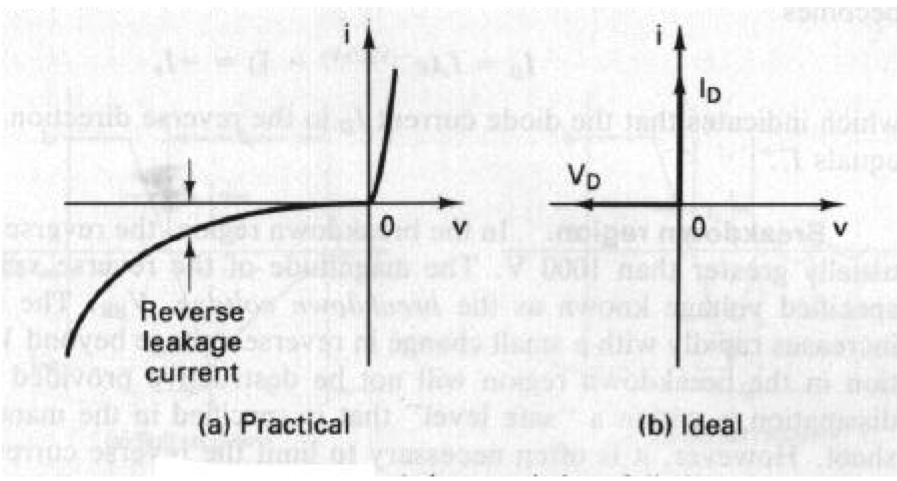

Figure (2): *v-i* characteristics of a diode

The *v-i* characteristics can be expressed by the Schockley Diode Equation as

$$I_D = I_s\left(e^{\frac{V_D}{nV_T}} - 1\right)$$

where

I_D = current through the diode, (A)

V_D = diode voltage with anode positive with respect to cathode, (V).

I_s = leakage (reverse saturation) current, typically in the range of 10^{-6} to 10^{-15} A.

n = empirical constant known as emission coefficient or ideality factor, $1 \le n \le 2$. Note that n depends on the material and physical construction of the diode. For **germanium** diodes $n=1$; for **silicon** diodes $n=2$.

V_T = constant called *thermal voltage* and is determined from

$$V_T = \frac{kT}{q}$$

where

q : is the electron charge $=1.6022x10^{-19}$ coulomb (C)
T : is the absolute temperature in Kelvin (K=273+C^{o})
k: is Boltzmann's constant = $1.3806x10^{-23}$ J/K.

Thus

$$V_T = \frac{1.3806\, x 10^{-23}\, x(273+25)}{1.6022\, x 10^{-19}} = 2.8\ mV$$

The diode *v-i* characteristics shown in figure (2a) can be divided into three regions, as explained below.

Forward-biased region **($V_D > 0$)**

In this region, the diode current I_D is very small if the diode voltage V_D is less than a specified value V_{TD} (typically 0.7 V). V_{TD} is called the *threshold* voltage or the *cut-in* voltage or the *turn-on* voltage. The diode conducts fully if V_D is higher than V_{TD}. Thus the *threshold voltage* V_{TD} is the voltage at which the diode conducts fully.

Example

Consider a diode of with $V_D = 0.1\ V$; $n=1$; and $V_T = 25.8\ mV$. Find the diode current.

$$I_D = I_s(e^{\frac{V_D}{nV_T}} - 1)$$

$$I_D = I_s(e^{\frac{0.1}{1x0.0258}} - 1) = I_s(48.23 - 1)$$

This can be approximated by

$$I_D \approx 48.23 I_s$$

This approximation produces an error of $\frac{1}{48.23} x 100 = 2.1\%$. If this is acceptable, then for $V_D > 0.1\ V$, which is usually the case, and $I_D >> I_s$ we can write

$$I_D = I_s(e^{\frac{V_D}{nV_T}} - 1) \approx I_s e^{\frac{V_D}{nV_T}}$$

Reverse-biased region **($V_D < 0$)**

In the reverse-biased region if V_D is negative and $|V_D| >> V_T$, which occurs for $V_D < -0.1\ V$, the exponential term becomes negligible and the diode current is approximated as

$$I_D = I_s(e^{\frac{V_D}{nV_T}} - 1) \approx -I_s$$

This means that the diode current I_D in the reverse direction is constant and equals I_s.

Breakdown Region

In this region the reverse voltage is high, normally greater than 1000 V. The magnitude of the reverse voltage exceeds a specified voltage known as the *breakdown voltage,*V_{BR}. The reverse current increases rapidly with a small change in the reverse voltage beyond the V_{BR}. It is often necessary to limit the power dissipation within a permissible value to avoid damage.

Example 1

The forward voltage drop of a power diode is $V_D = 1.2\ V$ at $I_D = 300\ A$. Assuming that n=2 and $V_T = 25.8\ mV$, find the saturation current I_s.

Solution: We have

$$I_D = I_s\left(e^{\frac{V_D}{nV_T}} - 1\right)$$

or

$$300 = I_s\left(e^{\frac{1.2}{2x0.0258}} - 1\right)$$

which gives

$$I_s = 2.38371x10^{-8}\ A$$

Reverse Recovery Characteristics

When a diode is in a forward conduction mode and then its forward current is reduced to zero, the diode continues to conduct due to minority carriers stored in the *pn*-junction and the bulk semiconductor material. The minority carriers require a certain time to be neutralised. This time is called the *reverse recovery time*, t_{rr}, of the diode.

Figure (3) shows the reverse recovery characteristics of junction diodes.

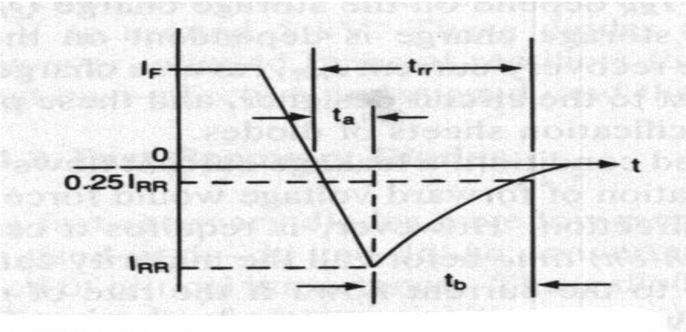

Figure (3)
Reverse Recover Characteristics: Soft recovery

Reverse Recovery Time (t_{rr})

Time interval between the instant the current passes through initial zero crossing during the changeover from forward conduction to reverse blocking condition and the instant when the reverse current decays to 25% of its maximum (peak) value, I_{RR}.

It consists of two components, t_a and t_b. Thus

$$t_{rr} = t_a + t_b$$

The component t_a is due to charge storage in the depletion region of the junction. It represents the time between the crossing and the peak reverse current. The component t_b is due to charge storage in the bulk semiconductor material. It represents the time between peak reverse current and when it decays to 25%.

The ratio $\frac{t_b}{t_a}$ is known as the *softness factor*. Thus

$$SF = \frac{t_b}{t_a}$$

The peak reverse current can be expressed in reverse di/dt as

$$I_{RR} = \frac{di_D}{dt}$$

Reverse Recovery Charge (Q_{RR})

The amount of charge carriers that flow across the diode in the reverse direction due to changeover from forward conduction to reverse blocking condition. Its value is determined from the area enclosed by the path of the reverse recovery current. Thus the storage charge is approximately

$$Q_{RR} \cong \frac{1}{2} I_{RR}\, t_a + \frac{1}{2} I_{RR}\, t_b = \frac{1}{2} I_{RR}\, t_{rr}$$

Hence

$$I_{RR} = \frac{2Q_{RR}}{t_{rr}}$$

Using

$$I_{RR} = \frac{di_D}{dt}$$

gives

$$t_{rr}t_a = \frac{2Q_{RR}}{di_D / dt}$$

Since usually $t_a > t_b$, then $t_{rr} \approx t_a$, and

$$t_{rr} \cong \sqrt{\frac{2Q_{RR}}{di_D / dt}}$$

and

$$I_{RR} \cong \sqrt{2Q_{RR}\frac{di_D}{dt}}$$

Example 2

The reverse recover time of a diode is $t_{rr} = 3\ \mu s$ and the rate of fall of the diode current is $\frac{di_D}{dt} = 30\ A / \mu s$. Determine

(a) the storage charge Q_{RR}; and

(b) the peak reverse current I_{RR}.

Solution

From $t_{rr} \cong \sqrt{\frac{2Q_{RR}}{di_D / dt}}$ we compute the storage charge as

$$Q_{RR} = \frac{1}{2}\frac{di_D}{dt}t_{rr}^2 = 0.5x30\ A / \mu s\ x \left(3x10^{-6}\right)^2 = 135\ \mu C$$

$$I_{RR} \cong \sqrt{2Q_{RR}\frac{di_D}{dt}} = \sqrt{2x135x10^{-6}\ x30\,x10^6} = 90\ A$$

Diode Circuits

In the following analysis, we assume that the reverse recover time and the forward voltage drop are negligible, i.e. $t_{rr} = 0$ and $V_D = 0$.

Diodes With RC Loads

Consider the diode circuit with an RC load shown in Figure (4).

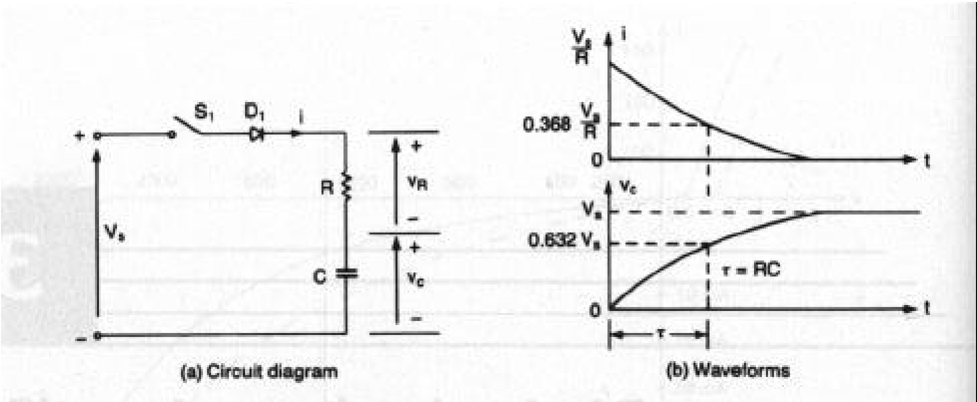

Figure (4): RC circuit

When switch S_1 is closed at $t=0$, the voltage relationship is derived as

$$V_s = v_R + v_C$$

$$= v_R + \frac{1}{C}\int i\,dt + v_C(t=0)$$

$$v_R = R\,i$$

Laplace Transform of both sides gives:

$$\frac{V_s}{s} = R\,I(s) + \frac{1}{Cs}I(s)$$

or

$$I(s) = \frac{V_s}{R(s + \frac{1}{RC})}$$

Inverse Laplace Transform gives

$$i(t) = \frac{V_s}{R} e^{-\frac{1}{RC}t}$$

The voltage across the capacitor is obtained as

$$v_C(t) = \frac{1}{C}\int_0^t i\,dt = V_s\left(1 - e^{-\frac{1}{RC}t}\right)$$

It is obvious from this relationship that the time constant of an RC load is

$$\tau = RC$$

The rate of change of the capacitor voltage is

$$\frac{dv_C}{dt} = \frac{V_s}{RC}e^{-\frac{1}{RC}t}$$

The initial rate of change of the capacitor voltage (at t=0) is obtained as

$$\left.\frac{dv_C}{dt}\right|_{t=0} = \frac{V_s}{RC}$$

The capacitor voltage and current responses are shown in Figure (4).

Diodes With RL Loads

Consider the diode circuit with an *RL* load shown in Figure (5).

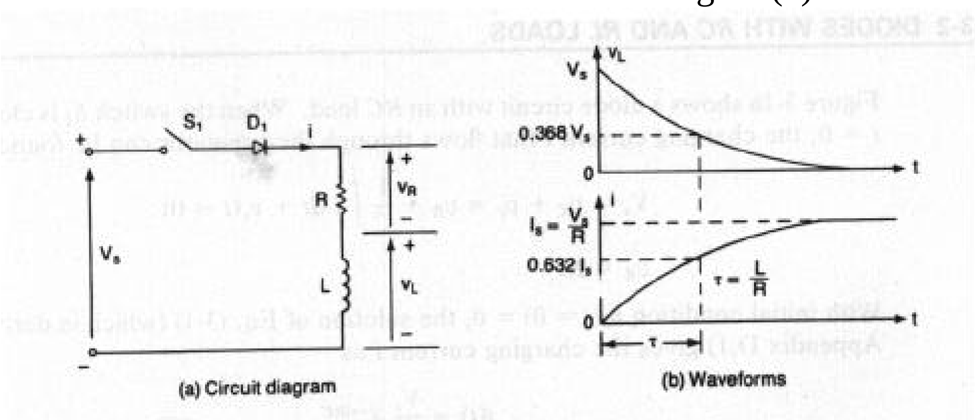

Figure (5): An RL circuit

When switch S_1 is closed at $t=0$, the current I through the inductor is expressed as

$$V_s = v_R + v_L$$

$$= v_R + L\frac{di}{dt}$$

$$v_R = R\,i$$

Thus for *i(0)=0*, the current through the conductor is obtained as

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

The voltage across the inductor is obtained as

$$v_L(t) = L\frac{di}{dt} = V_s e^{-\frac{R}{L}t}$$

It is obvious from this relationship that the time constant of an *RL* load is

$$\tau = \frac{L}{R}$$

The rate of change of the current is obtained as

$$\frac{di}{dt} = \frac{V_s}{L} e^{-\frac{R}{L}t}$$

The initial rate of change of the current (at *t=0*) is obtained as

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_s}{L}$$

The inductor voltage and current responses are shown in Figure (5).

Analysis

For $t >> \frac{L}{R} = \tau$, $v_L \rightarrow 0$ and $i \rightarrow I_s = \frac{V_s}{R}$.

If S_1 is opened, then the energy stored in the inductor is

$$E_L = 0.5L\ i^2$$

This energy is transformed into a high reverse voltage across the switch. In this case the diode D_1 is likely to be damaged in this process. To overcome this problem a diode commonly known as a *freewheeling diode* is connected across an inductive load as shown in Figure (6).

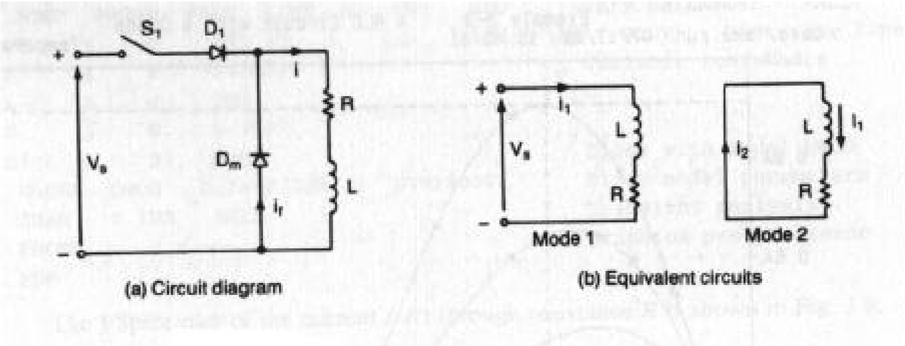

Figure (6): Circuit with a Freewheeling Diode

Freewheeling Diodes

Consider the RL circuit shown in Figure (6). If the switch S_1 is closed for time t_1, a current is established through the load. If then the switch is opened, a path must be provided for the current in the inductive load.

This is normally done by connecting a diode called a *freewheeling diode* D_m as shown in Figure (6).

The circuit operation can be divided into two modes:

Mode 1:
This mode begins when the switch is closed at $t=0$ until the switch is opened. This duration is denoted by t_1. During this mode, the diode current is

$$i_1(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

When the switch is opened at $t=t_1$, the current becomes

$$I_1 = i_1(t = t_1) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t_1}\right)$$

If t_1 is sufficiently long, the current reaches a steady-state value of

$$I_s = \frac{V_s}{R}$$

Mode 2:
This mode begins when the switch is opened and load current starts to flow in the freewheeling diode D_m. The current through the freewheeling diode is found from

$$0 = L\frac{di_2}{dt} + R\,i_2$$

with the initial condition $i_2(t=0) = I_1$. The solution to this equation gives

$$i_2(t) = I_1 e^{-\frac{R}{L}t}$$

and this current decays exponentially to zero at $t = t_2$ provided that $t_2 >> L/R$. The currents responses are shown in Figure (6).

Example 3
A diode circuit is shown in Figure (7) with $R = 44\ \Omega$ and $C = 0.1\ \mu F$. The capacitor has an initial voltage $V_0 = 220\ V$. If switch S_1 is closed at $t=0$, determine:

(a) the peak diode current
(b) the energy dissipated in the resistor R
(c) the capacitor voltage at $t = 2\ \mu s$.

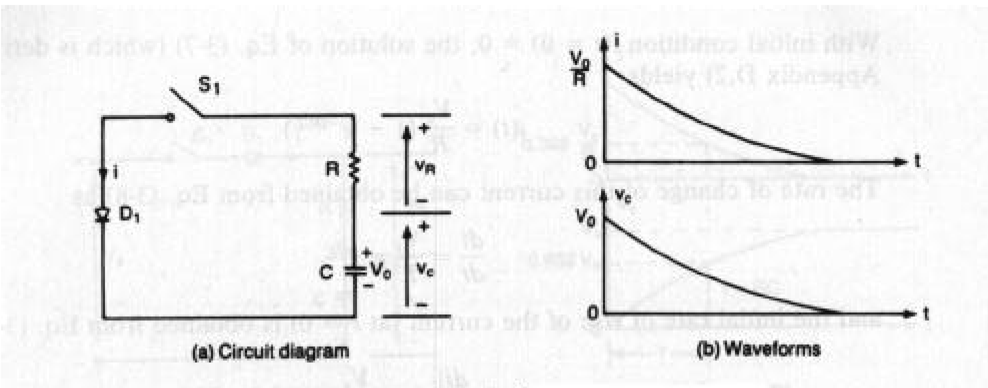

Figure (7): An RC circuit

Solution

The waveforms are shown in Figure (7).

(a) From the relationship

$$i(t) = \frac{V_s}{R} e^{-\frac{1}{RC}t}$$

the peak value of the diode current is obtained by using $V_s = V_0$. Thus

$$I_p = \frac{V_0}{R} = \frac{220}{44} = 5A$$

(b) The energy dissipated is

$$E = 0.5CV_0^2 = 0.5x0.1x10^{-6}x220^2$$

or

$$E = 2.42\ mJ$$

(c) For $RC = 44x0.1 = 4.4\mu s$ and $t = t_1 = 2\ \mu s$, the capacitor voltage is

$$v_C(t = 2\mu s) = V_0 e^{-\frac{1}{RC}t}$$

or

$$= 220xe^{-\frac{2}{4.4}} = 139.64\ V$$

Diodes With LC Load

A diode circuit with an *LC* load is shown in Figure (8).

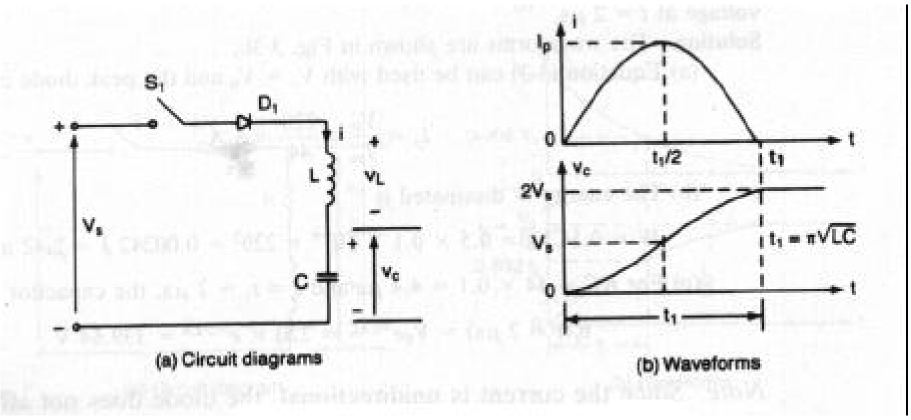

Figure (8): *LC* Circuit

When the switch S_1 is closed at $t=0$, the charging current i of the capacitor is expressed as

$$V_s = L\frac{di}{dt} + \frac{1}{C}\int i\,dt + v_C(t=0)$$

with initial conditions $i(t=0)=0$ and $v_C(t=0)=0$. Laplace transform gives

$$\frac{V_s}{s} = L\,s\,I(s) + \frac{1}{Cs}I(s)$$

or

$$I(s) = \frac{V_s}{L\left(s^2 + \frac{1}{LC}\right)}$$

Inverse Laplace Transform gives

$$i(t) = V_s\sqrt{\frac{C}{L}}\,\sin\omega t = I_p\sin\omega t$$

where $\omega = \frac{1}{\sqrt{LC}}$.

The peak current is

$$I_p = V_s\sqrt{\frac{C}{L}}$$

The rate of rise of the current is obtained from

$$i(t) = V_s\sqrt{\frac{C}{L}}\,\sin\omega t = I_p\sin\omega t$$

$$\frac{di}{dt} = \frac{V_s}{L}\cos\omega t$$

From this, the initial rate rise of the current (at $t=0$) is obtained as

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_s}{L}$$

The voltage across the capacitor is derived as

$$v_C(t) = \frac{1}{C}\int_0^t i dt = V_s(1 - \cos\omega t)$$

At time $t = t_1 = \pi\sqrt{LC}$, the diode current *i* decays to zero and the capacitor is charged to $2V_s$. The waveforms for the voltage v_L and the current *i* are shown in Figure (8).

Example 4
A diode circuit with an *LC* load is shown in Figure (9a) with the capacitor having an initial voltage *220 V*. The capacitance is $C = 20\ \mu F$ inductance is $L = 80\ \mu H$. If switch S_1 is closed at *t=0*, determine:

(a) the peak current through the diode.
(b) the conduction time of the diode.
(c) the steady-state capacitor voltage.

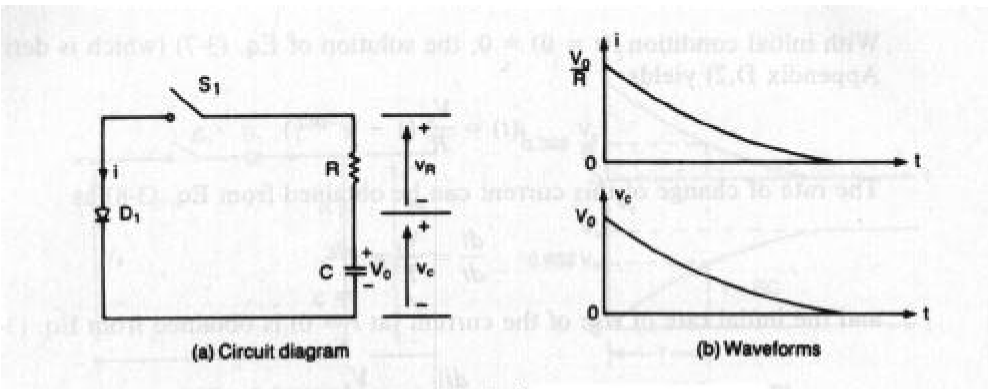

Figure (9a,b); fig 3.3

Solution (a)
Using Kirchoff's voltage law gives

$$L\frac{di}{dt} + \frac{1}{C}\int i\ dt + v_C(t = 0) = 0$$

With initial conditions i(t=0)=0 and $v_C(t = 0) = -V_0$, the current *i* is obtained as

$$i(t) = V_0 \sqrt{\frac{C}{L}} \sin \omega t$$

where

$$\omega = \frac{1}{\sqrt{LC}} = \frac{10^6}{\sqrt{20 x 80}} = 25,000 \ rad \ / \ s$$

The peak current is therefore obtained as

$$I_p = V_0 \sqrt{\frac{C}{L}} = 220 \sqrt{\frac{20}{80}} = 110 \ A$$

Solution (b)

At $t = t_1 = \pi \sqrt{LC}$, the diode current becomes zero and the conduction time is

$$t_1 = \pi \sqrt{LC} = \pi \sqrt{20 x 80} \ x10^{-6} = 125.66 \ \mu s$$

(c) The capacitor voltage can be obtained as

$$v_C(t) = \frac{1}{C} \int i \ dt - V_0 = -V_0 \cos \omega t$$

or

$$v_C(t = t_1 = 125.66 \mu s) = -220 \cos \pi = 220 \ V$$

The waveforms are shown in Figure (9b).

Diodes with RLC Loads

A diode circuit with an *RLC* load is shown in Figure (10).

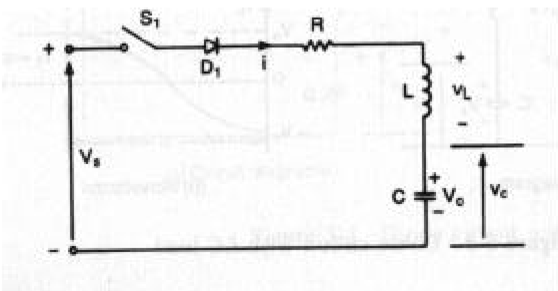

Figure (10): An RLC Circuit

When the switch S_1 is closed at $t=0$, the charging current i is expressed as

$$V_s = L\frac{di}{dt} + \frac{1}{C}\int i\,dt + v_C(t=0) + Ri$$

with initial conditions *i(t=0)* and $v_C(t=0) = V_0$.

Differentiating and dividing by *L* gives

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

Laplace transform gives

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

The roots of this equation (called characteristic equation) are obtained as

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Define:

$\alpha = \frac{R}{2L}$ (this is called the damping factor) and

$\omega_n = \frac{1}{\sqrt{LC}}$ (this is called the natural frequency).

The roots of the characteristic equation may now be expressed as

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$

Thus three possible solutions for the current are obtained as

Solution 1:

$\alpha = \omega_n$. This means that the roots are real and equal ($s_1 = s_2$) and the current response is

$$i(t) = (A_1 + A_2 t)e^{s_1 t}$$

This response is called *critically damped*.

Note that the constants A_1 and A_2 are determined from the initial conditions of the circuit.

Solution 2:
$\alpha > \omega_n$. This means that the roots are real but different in magnitude and the response is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

This response is called *overdamped*.

Solution 3:
$\alpha < \omega_n$. This means that the roots are complex-conjugate ($s_{1,2} = -\alpha \pm j\omega_d$) and the response is

$$i(t) = e^{-\alpha t}\left(A_1 \cos \omega_d t + A_2 \sin \omega_d t\right)$$

This response is called *damped sinusoidal*.

Note that ω_d is called the damped frequency and given as

$$\omega_d = \omega_n \sqrt{1 - \left(\frac{\alpha}{\omega_n}\right)^2}$$

If we define the damping ratio as

$$\eta = \frac{\alpha}{\omega_n} = \frac{RC}{2\sqrt{L}}$$

then

$$\omega_d = \omega_n \sqrt{1 - \eta^2}$$

Example 5
A diode circuit with an *RLC* load is shown in Figure (11). The source voltage is $V_s = 220\ V$. The capacitance is $C = 0.05\ \mu F$ inductance is $L = 2\ mH$ and the

resistance is $R = 160\ \Omega$. The initial value of the capacitor voltage is $V_0 = 0$. If switch S_1 is closed at $t=0$, determine:

(a) an expression for the current *i(t)* and sketch *i(t)*.
(b) the conduction time of the diode.

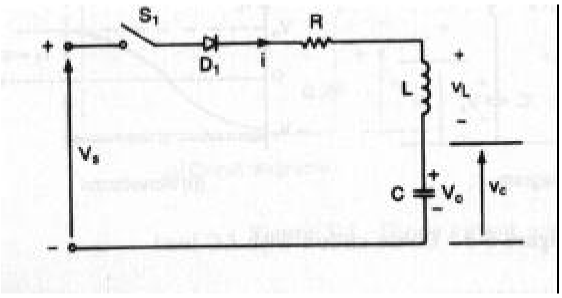

Figure (11): An RLC Circuit

Solution
The damping factor and the natural frequency are first obtained as

$$\alpha = \frac{R}{2L} = \frac{160 x 10^3}{2 x 2} = 40{,}000\ rad\ /\ s$$

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2x10^{-3} x 0.05 x 10^{-6}}} = 10^5\ rad\ /\ s$$

The damping ratio and the damped frequency are then determined as

$$\eta = \frac{\alpha}{\omega_n} = \frac{40{,}000}{100{,}000} = 0.4$$

$$\omega_d = \omega_n \sqrt{1-\eta^2} = 10^5 \sqrt{1-0.16} = 91{,}652\ rad\ /\ s$$

Since $\alpha < \omega_n$, the current response is *underdamped* and given as

$$i(t) = e^{-\alpha t}\left(A_1 \cos \omega_d t + A_2 \sin \omega_d t\right)$$

At *t=0* we have

$$i(t=0) = 0 = 1\left(A_1 \cos 0 + A_2 \sin 0\right) = A_1$$

Thus the solution is

$$i(t) = e^{-\alpha t}\left(A_2 \sin \omega_d t\right)$$

Taking the first derivative gives

$$\left.\frac{di}{dt}\right|_{t=0} = e^{-\alpha t}\left(A_2 \cos \omega_d t - \alpha A_2 \sin \omega_d t\right)\Big|_{t=0}$$

or

$$\left.\frac{di}{dt}\right|_{t=0} = \omega_d A_2 = \frac{V_s}{L}$$

From this we evaluate A_2 as

$$\left.\frac{di}{dt}\right|_{t=0} = A_2 = \frac{V_s}{\omega_d L} = \frac{220\,x\,1000}{91{,}652\,x\,2} = 1.2$$

The expression for the current is therefore obtained as

$$i(t) = 1.2\ \sin\left(91{,}652t\right)e^{-40{,}000\,t}\ A$$

A sketch of the current waveform is shown in Figure (12).

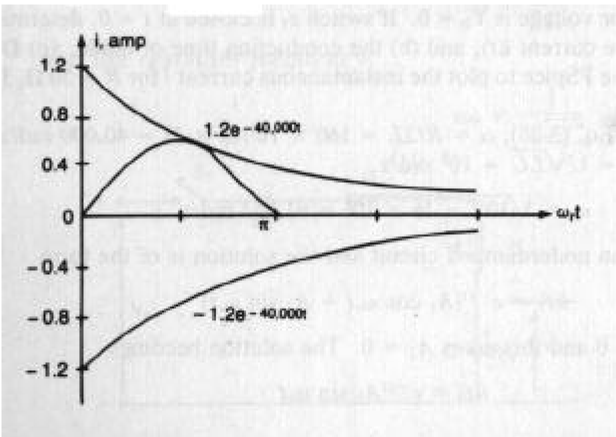

Figure (12)

(b) The conduction time t_1 of the diode is obtained when the current i falls to zero. This happens when $\sin(91{,}652t_1) = 0$. Or when

$$91{,}652 t_1 = \pi$$

From this

$$t_1 = \frac{\pi}{91{,}652} = 34.27 \ \mu s$$