Single-Phase Half-Wave Rectifiers

A rectifier is a circuit that converts an ac signal into a unidirectional signal. A single-phase half-way rectifier is the simplest type. Although it is not widely used in industry, it helps undertand the principle of rectifier operation.

Resistive Load

A circuit digram with a resistive load is shown in Figure (1).

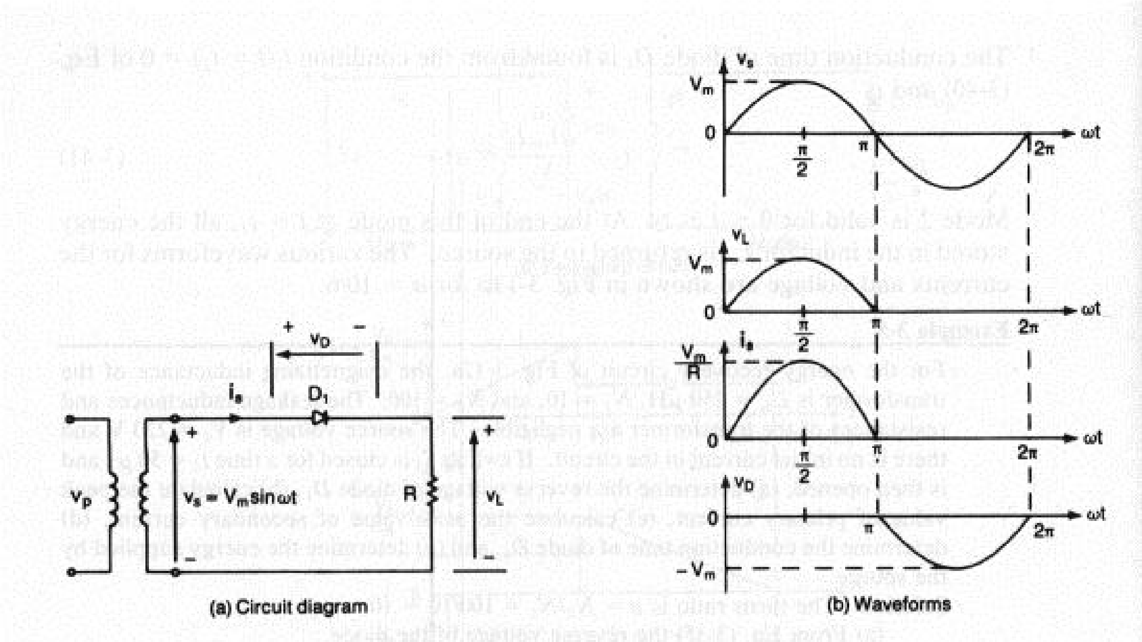

Figure (1): A Single-Phase Half-Wave Rectifier

During the positive half cycle of V_S

- diode D_1 conducts
- $V_L = V_S$.

During the negative half cycle of V_S

- diode D_1 is in a *blocking condition*
- $V_L = 0$.

Performance Parameters

The performance of a rectifier is normally evaluated in terms of the following parameters
The *average* value of the output (load) voltage, V_{dc}.
The *average* value of the output (load) current, I_{dc}.
The output (load) *dc* power

$$P_{dc} = V_{dc} I_{dc}.$$

The *rms* value of the output (load) voltage, V_{rms}.
The *rms* value of the output (load) current, I_{rms}.
The output (load) ac power

$$P_{ac} = V_{rms} I_{rms}$$

The efficiency (or rectification ratio) of a rectifier is defined as

$$\xi = \frac{P_{dc}}{P_{ac}}$$

The output voltage can be viewed as being composed of two components

(1) the *dc* value; and
(2) the *ac* component or ripple.

The *rms* value of the *ac* component of the output voltage is

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2}$$

The *form factor*, which is a measure of the shape of the output voltage, is defined as

$$FF = \frac{V_{rms}}{V_{dc}}$$

The ripple factor, which a measure of the ripple content, is defined as

$$RF = \frac{V_{ac}}{V_{dc}}$$

It therefore follows that

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{FF^2 - 1}$$

The *transformer utilization factor* is defined as

$$TUF = \frac{P_{dc}}{V_s I_s}$$

where V_s and I_s are the *rms* voltage and *rms* current of the transformer secondary, respectively.

Consider now waveforms shown in Figure (2).

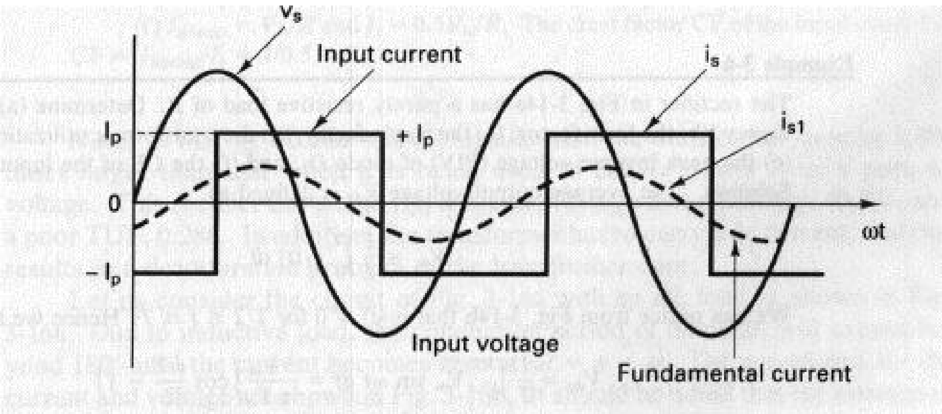

Figure (2): Waveorms of Input Voltage and Current

where
v_s is the sinusoidal voltage
i_s is the instantaneous input current
i_{s1} is the fundamental component of i_s.

Let Φ be the *displacement angle* between the fundamental components of the current and voltage. Then the *displacement factor* is defined as

$$DF = cos\ \Phi$$

The harmonic factor of the input current is defined as

$$HF = \sqrt{\left(\frac{I_s^2 - I_{s1}^2}{I_{s1}^2}\right)} = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1}$$

where I_{s1} is the fundamental component of the input current I_s. Both of these are expressed in terms of *rms*.

The input *power factor* is defined as

$$PF = \frac{V_s I_{s1}}{V_s I_s} cos\ \Phi = \frac{I_{s1}}{I_s} cos\ \Phi$$

Crest Factor CF, which is a measure of the peak input current $I_{s(peak)}$ as compared to its *rms* value I_s, is defined as

$$CF = \frac{I_{s(peak)}}{I_s}$$

Notes

1. The harmonic factor HF is a measure of the distortion of a waveform and is also known as *total harmonic distortion* (THD).

2. If the input current is purely sinusoidal, then $I_s = I_{s1}$ and PF=DF. Also the displacement angle Φ becomes the impedance angle $\Phi = tan^{-1}\left(\frac{\omega L}{R}\right)$ for an RL load.

3. Displacement factor DF is often known as *displacement power factor* (DPF).
4. An ideal rectifier should have 100% efficiency, V_{ac}=0, RF=0, TUF=1, HF=THD=0, and PF=DPF=1.

Example
The Rectifier in Figure (1) has a purely resistive load of R. Determine:
(a) the efficiency,
(b) the form factor,
(c) the ripple factor,
(d) the transformer utilisation factor, and
(e) the crest factor of the input current.

Solution
The average output voltage V_{dc} is defined as

$$V_{dc} = \frac{1}{T}\int_0^T v_L(t)dt$$

From Figure (1), it is clear that

$$v_L = 0 \text{ for } T/2 \le t \le T$$

Hence

$$V_{dc} = \frac{1}{T}\int_0^{T/2} v_m sin(\omega t)dt = \frac{-V_m}{\omega T}\left(cos\frac{\omega T}{2} - 1\right)$$

Since $f = 1/T$ and $\omega = 2\pi f$, we obtain

$$V_{dc} = \frac{V_m}{\pi} = 0.318\ V_m$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.318\ V_m}{R}$$

Note that the presence of this current means that the transformer will have to carry a *dc* current which results in *dc* saturation of the transformer core.

The rms value of a periodic waveform is defined as

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v_L^2(t)\,dt}$$

Since $v_L = V_m \sin \omega t$ for $0 \le t \le T/2$, the rms value of the output voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T/2} \left(V_m \sin \omega t\right)^2 dt} = 0.5V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.5V_{rms}}{R}$$

Thus

$$P_{dc} = V_{dc} I_{dc} = \frac{\left(0.318V_m\right)^2}{R}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{\left(0.5V_m\right)^2}{R}$$

(a) The efficiency

$$\xi = \frac{P_{dc}}{P_{ac}} = \frac{\left(0.318V_m\right)^2}{\left(0.5V_m\right)^2} = 40.5\%,$$

This is considered as low efficiency.

(b) The form factor

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{\left(0.5V_m\right)}{\left(0.318V_m\right)} = 157\%,$$

(c) The ripple factor

$$RF = \sqrt{FF^2 - 1} = \sqrt{1.57^2 - 1} = 121\%,$$

This is considered high for practical application.

(d) The *rms* voltage of the transformer secondary

$$V_s = \sqrt{\frac{1}{T}\int_0^T \left(V_m \sin \omega t\right)^2 dt} = \frac{V_m}{\sqrt{2}} = 0.707V_m,$$

The rms value of the transformer secondary current is the same as that of the load

$$I_s = \frac{0.5V_m}{R},$$

The *volt-ampere* rating of the transformer is

$$VA = V_s I_s = \frac{V_m}{\sqrt{2}}\frac{V_m}{2R} = \frac{V_m^2}{2\sqrt{2}R},$$

The *transformer utilization factor* is

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{\dfrac{(0.318V_m)^2}{R}}{\dfrac{V_m^2}{2\sqrt{2}R}} = 0.286,$$

This is a poor utilization factor. The reciprocal of the TUF is

$$\frac{V_s I_s}{P_{dc}} = \frac{1}{TUF} = \frac{1}{0.286} = 3.496$$

This means that the transformer must be 3.496 times larger than when it is being used from a pure *ac* voltage.

(e) $I_{s(peak)} = \dfrac{V_m}{R}$ and $I_s = \dfrac{V_m}{2R}$. Thus the crest factor of the input current is

$$CF = \frac{I_{s(peak)}}{I_s} = 2.$$

Half-Wave Rectifier: Resistive and Inductive Load

Consider the circuit shown in Figure (3a).

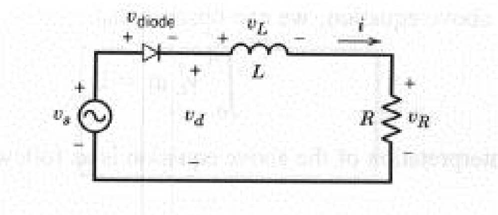

Figure (3a): Rectifier with an *RL* load

The voltage and current wave forms are shown in Figures (3b,c,d).

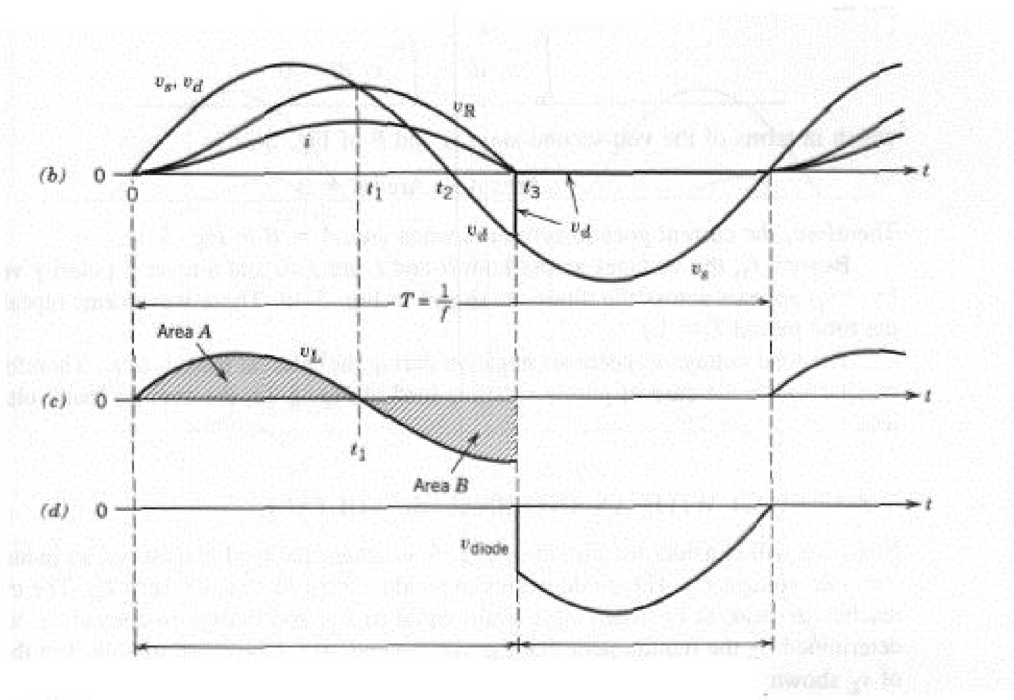

Figure (3b,c,d): Voltage and current waveforms

At $t<0$, $i=0$ and v_s is -ve.

At $t=0$, diode becomes forward biased and i begins to flow. The diode can the be replaced by a short as shown in Figure (3,e).

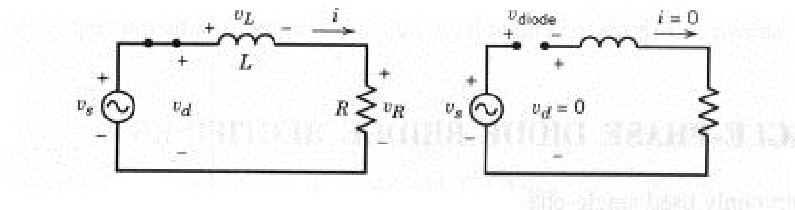

Figure (3e): Equivalent circuit for the rectifier of fig (3a)

The current is governed by

$$v_s = Ri + L\frac{di}{dt}$$

The resulting currents and voltages are shown in Figures (3b,c).

For $0 < t \leq t_1$
- $v_s > v_R$, and $v_L = v_s - v_R$ is +ve.
- i builds up.
- inductor stored energy increases.

For $t_1 < t \le t_2$:
• v_L becomes -ve.
• i begins to decrease.

For $t_2 < t \le t_3$:
• v_s becomes -ve.
• v_d becomes -ve.
• i is still +ve.
• diode still conducts because of the inductor stored energy.

To find the instant when $i=0$ and the diode stops conducting we use the following analysis

$$v_L = L\frac{di}{dt}$$

or

$$\frac{1}{L}v_L dt = di$$

Integrating both sides between $t=0$ and $t = t_3$, and using the fact that $i(0)=i(t_3)=0$, gives

$$\frac{1}{L}\int_0^{t_3} v_L dt = \int_{i(0)}^{i(t_3)} di = i(t_3) - i(0) = 0$$

This means that

$$\int_0^{t_3} v_L dt = 0$$

or

$$\int_0^{t_1} v_L dt + \int_{t_1}^{t_3} v_L dt = 0$$

or

Area A - Area B = 0

This means that $i=0$ when

Area A = Area B.

For $t > t_3$:

• $v_L = v_R = 0$.
• reverse polarity voltage appears across the diode.

The waveforms repeat themselves strating from $t=T=1/f$.

Single-Phase Full-Wave Rectifier

Consider the full-wave rectifier shown in Figure (4a). The transformer is centre-tapped. Each half of the transformer and its associated diode acts as half-wave rectifier. The output of a full-wave rectifier is shown in Figure (4b). Note that since there is no dc current flowing through the transformer, there is no dc saturation problem of the trabsformer core.

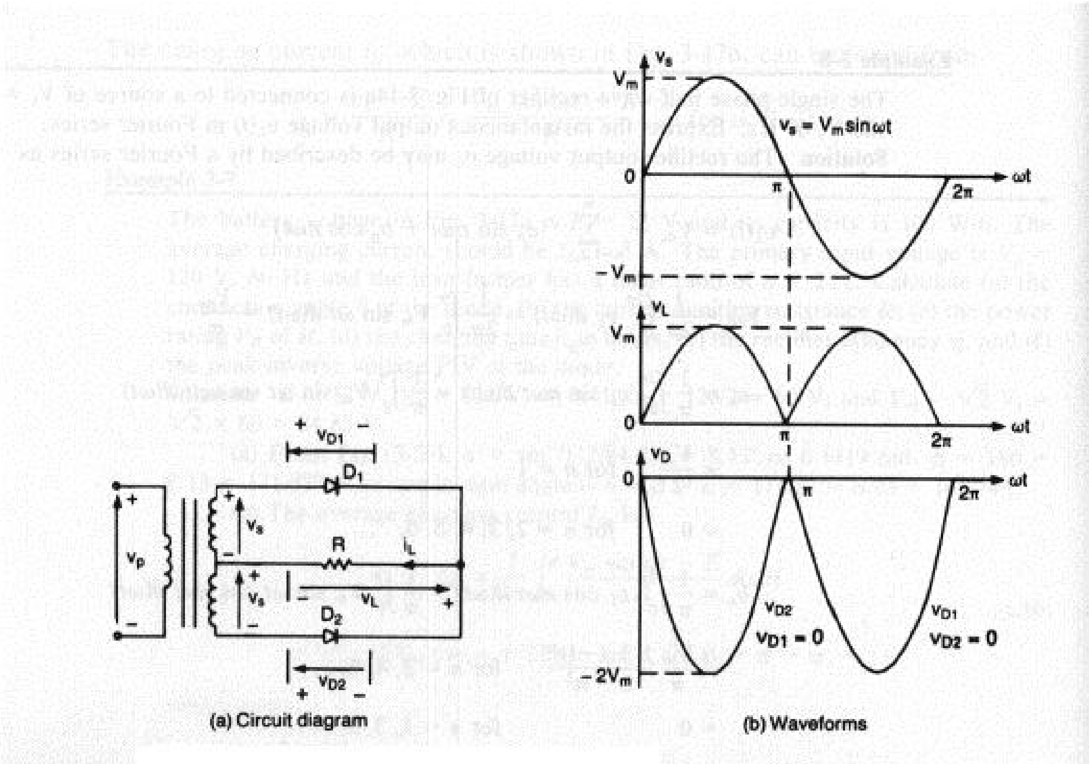

Figure (4): A full-wave rectifier circuit with centre-tapped transformer

The average output voltage is

$$V_{dc} = \frac{2}{T}\int_0^{T/2} V_m \sin\omega t \, dt = \frac{2V_m}{\pi} = 0.6366 V_m$$

An alternative to using a centre tapped transformer, it is possible to use four diodes as shown in Figure (5a). During the positive half-cycle of the input voltage, the power is supplied to the load through diodes D_1 and D_2. During the negative half-cycle, the power is supplied to the load through diodes D_3 and D_4. The waveform of the output voltage is shown in Figure (5b), which is similar to that of Figure (4b).

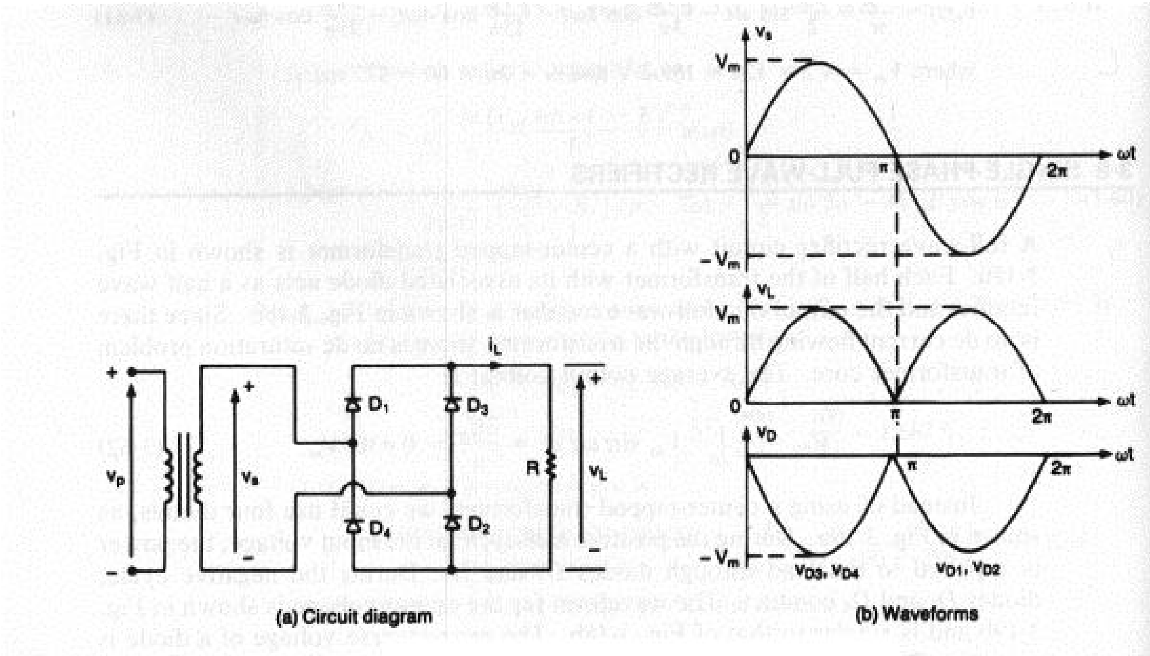

Figure (5): A full-wave rectifier circuit with four diodes

The peak-inverse voltage of a diode is only V_m. The circuit of Figure (5a) is known as a "*bridge rectifier*" and is commonly used in industrial applications.

Example
If the rectifier in Figure (4a) has a purely resistive load of R, determine:
(a) the efficiency
(b) the form factor
(c) the ripple factor
(d) the transformer utilization factor
(e) the peak-inverse voltage of diode D_1.
(f) the CF of the input current.

Solution
The average output voltage is

$$V_{dc} = \frac{2}{T}\int_0^{T/2} V_m \sin\omega t\, dt = \frac{2V_m}{\pi} = 0.6366 V_m$$

The average load current is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.6366 V_m}{R}$$

The rms value of the output voltage is

$$V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} \left(V_m \sin\ \omega t\right)^2 dt} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

The rms value of the current is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707V_m}{R}$$

The output dc power is

$$P_{dc} = V_{dc} I_{dc} = \frac{(0.6366V_m)^2}{R}$$

The output ac power is

$$P_{ac} = V_{rms} I_{rms} = \frac{(0.707V_m)^2}{R}$$

(a) The efficiency is

$$\xi = \frac{P_{dc}}{P_{ac}} = \left(\frac{0.6366}{0.707}\right)^2 = 81\%$$

(b) The form factor is

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{0.707}{0.6366} = 1.11$$

(c) The ripple factor is

$$RF = \sqrt{FF^2 - 1} = \sqrt{1.11^2 - 1} = 48.2\%$$

(d) The rms voltage of the transformer secondary is

$$V_s = 0.707V_m$$

The rms value of the transformer secondary current is

$$I_s = \frac{0.5V_m}{R}$$

The VA rating of the transformer is

$$VA = 2V_s I_s = 2(0.707V_m)(\frac{0.5V_m}{R}) = \frac{0.707V_m^2}{R}$$

The transformer utilization factor is therefore

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{(0.6366V_m)^2 / R}{0.707V_m^2 / R} = \frac{0.6366^2}{0.707} = 57.32\%$$

(e) The peak reverse blocking voltage is

$$PIV = 2V_m$$

(f) The peak value of the transformer secondary current is

$$CF = \frac{I_{s,peak}}{I_{s,rms}} = \frac{V_m / R}{0.707V_m / R} = \sqrt{2}$$

Single-Phase Full-Wave Rectifier with RL Load

With a resistive load, the load current is identical in shape to the output voltage. In practice, most loads are inductive to a certain extent and the load current depends on the values of the load resistance R and the load inductance L. This is illustrated in Figure (6a). A battery of voltage E is added to develop generalised equations.

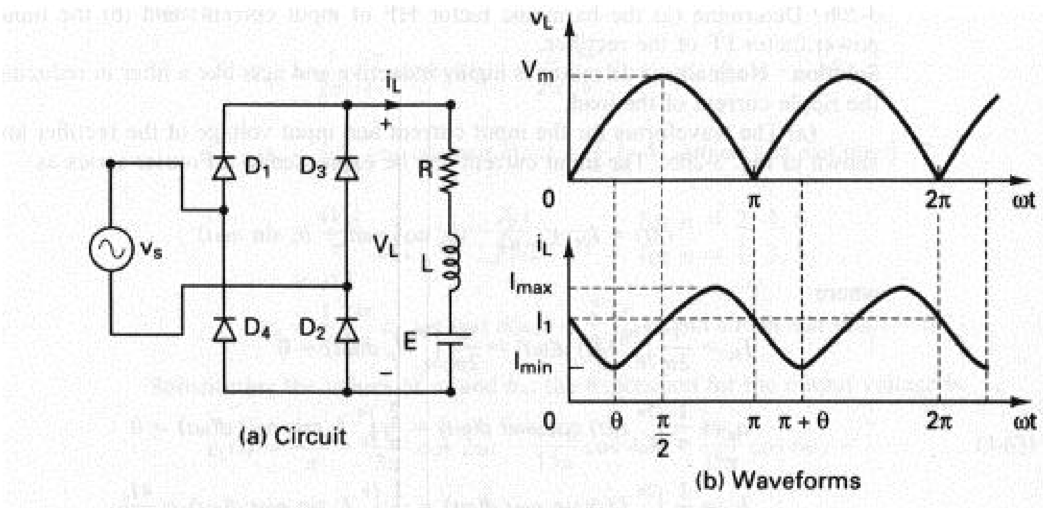

Figure (6): A full-wave rectifier circuit with *RL* load

If the input voltage is given as

$$v_s = V_m \sin \omega t = \sqrt{2}V_s \sin \omega t$$

then the load current i_L can be obtained from

$$L\frac{di_L}{dt} + Ri_L + E = \sqrt{2}V_s \sin \omega t$$

The solution to this equation is

$$i_L = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + A_1 e^{-\frac{R}{L}t} - \frac{E}{R}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Case 1: Continuous load current

The constant A_1 can be determined from the condition that at $\omega t = \pi$, $i_L = I_1$.
Thus

$$A_1 = \left(I_1 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin\theta\right)e^{\frac{R}{L}\frac{\pi}{\omega}}$$

Thus

$$i_L = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + \left(I_1 + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z}\sin\theta\right)e^{\frac{R}{L}\left(\frac{\pi}{\omega} - t\right)}$$

Using the initial condition $i_L(\omega t = 0) = I_1$ gives

$$I_1 = \frac{\sqrt{2}V_s}{Z}\sin\theta\,\frac{1 + e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}{1 - e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}} - \frac{E}{R}$$

Using this in i_L gives the following relationship

$$i_L = \frac{\sqrt{2}V_s}{Z}\left[\sin(\omega t - \theta) + \frac{2}{1 - e^{-\left(\frac{R}{L}\right)\left(\frac{\pi}{\omega}\right)}}\sin\theta e^{-\left(\frac{R}{L}\right)t}\right] - \frac{E}{R}$$

where $0 \le \omega \le \pi$ and $i_L \ge 0$.

The diode rms current can now be found as

$$I_{rms,diode} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_L^2\, d(\omega t)}$$

The rms output current can then be determined by combining the rms current of each diode as

$$I_{rms} = \sqrt{I^2_{ems,diode} + I^2_{ems,diode}} = \sqrt{2}I_{rms,diode}$$

The average diode current can also be found as

$$I_{a,diode} = \frac{1}{2\pi}\int_0^{\pi} i_L \, d(\omega t)$$

Case 2: discontinuous load current
The load current flows only during the period $\alpha \le \omega t \le \beta$. The diods start to conduct at $\omega t = \alpha$ where α is given by

$$\alpha = sin^{-1}\frac{E}{V_m}$$

At $\omega t = \alpha$, $i_L(\omega t) = 0$. Using this in the solution for i_L gives

$$A_1 = \left(\frac{E}{R} - \frac{\sqrt{2}V_s}{Z} sin(\alpha - \theta)\right) e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

Thus

$$i_L = \frac{\sqrt{2}V_s}{Z} sin(\omega t - \theta) + \left(\frac{E}{R} - \frac{\sqrt{2}V_s}{Z} sin(\alpha - \theta)\right) e^{\frac{R}{L}\left(\frac{\alpha}{\omega} - t\right)}$$

Using the condition $i_L(\omega t = \beta) = 0$ gives

$$\frac{\sqrt{2}V_s}{Z} sin(\beta - \theta) + \left[\frac{E}{R} - \frac{\sqrt{2}V_s}{Z} sin(\alpha - \theta)\right] e^{\left(\frac{R}{L}\right)\left(\frac{\alpha - \beta}{\omega}\right)} = 0$$

The value of β can be determined by an iterative method, starting from β=0 and increasing its value by a small amount until the left-hand becomes zero.

The rms diode current can be found as

$$I_{rms,diode} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i_L^2 \, d(\omega t)}$$

The average diode current can also be found as

$$I_{a,diode} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i_L \, d(\omega t)$$

Example
A single-phase full-wave rectifier is shown in figure (6a). It has $L = 6.5 mH$, $R = 2.5\ \Omega$, and $E = 10\ V$. The input current is $V_s = 120\ V$ at $60\ Hz$. Determine:

(a) the steady-state load current I_1 at $\omega t = 0$.
(b) the average diode current $I_{a,diode}$.
(c) the rms diode current $I_{rms,diode}$.
(d) the rms output current $I_{output,rms}$.

Solution
Assume that the load current is continuous.

$$\omega L = 2\pi fL = 2\pi(60)(0.0065) = 2.45\ \Omega$$

$$Z = \sqrt{2.5^2 + 2.45^2} = 3.5\ \Omega$$

$$\theta = tan^{-1}\left(\frac{2.45}{2.5}\right) = 44.43^o$$

(a) The steady-state load current at $\omega t = 0$ is

$$I_1 = 32.8\ A$$

Since this is not zero, the load current is continuous. Thus the assumption is correct.

(b) Using a numerical integration method gives

$$I_{a,diode} = 19.61\ A$$

(c) Using a numerical integration method gives

$$I_{rms,diode} = 28.5\ A$$

(d) The rs output current is

$$I_{rms} = \sqrt{2} I_{rms,diode} = \sqrt{2} x 28.5 = 40.3\ A$$

Single-Phase Half-Wave Rectifiers
with RL Load

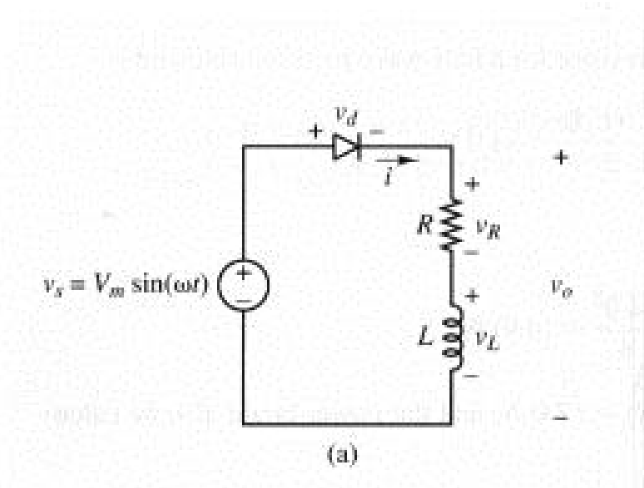

Figure (7)

The Kirchhoff's voltage law when the diode is in the forward-bias mode is

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

The solution to this equation is obtained as

$$i(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

where

$$\tau = \frac{L}{R} \equiv time\ constant$$

The constant parameter A is evaluated from the initial condition. As the initial current is zero before the diode started conducting, we have

$$i(0) = \frac{V_m}{Z}\sin(0 - \theta) + Ae^{0}$$

or

$$A = \frac{V_m}{Z}\sin\theta$$

Using this in the current equation gives

$$i(t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\frac{t}{\tau}}\right]$$

or

$$i(t)=\frac{V_m}{Z}\left[\sin(\omega t-\theta)+\sin(\theta)e^{-\frac{\omega t}{\omega\tau}}\right]$$

This relationship is valid only when *i(t)* is positive. If the current is negative then its actual value is zero.

When the source voltage is positive, the diode conducts and the voltage waveform of each element is shown in Figure (7b).

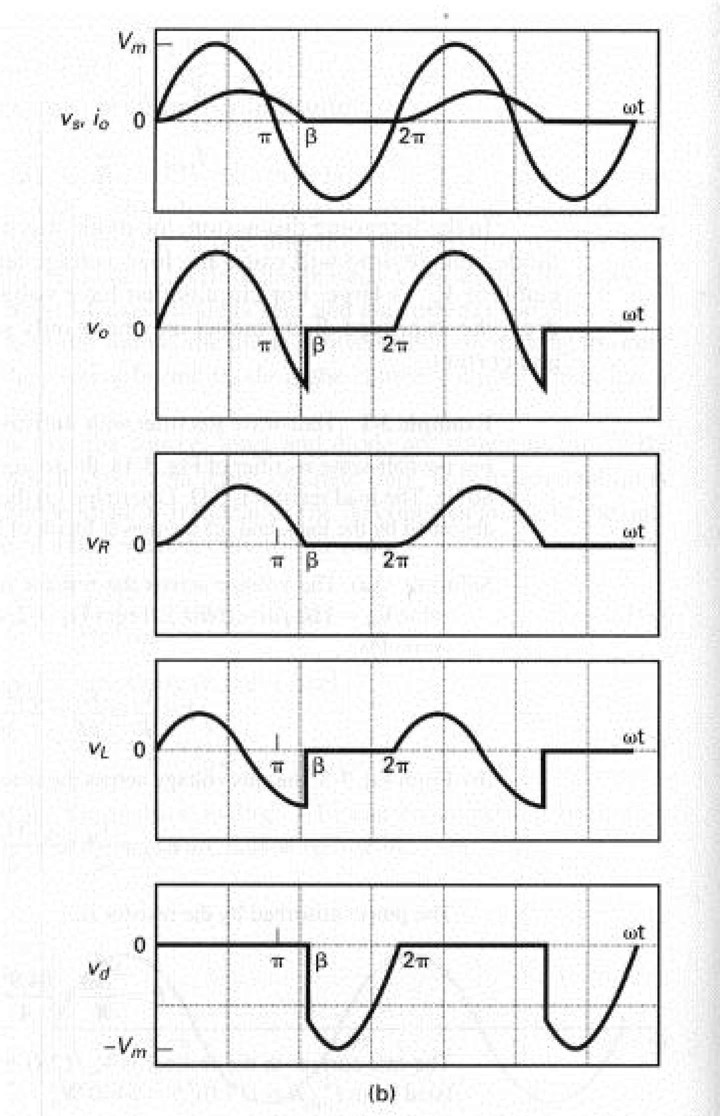

Figure (7b)

Note that

- the diode remains forward biased for $\beta > \pi$.
- the source voltage is negative for the last part of the conducting period.

• since $v_L = L\ di / dt$, the inductor voltage is negative when the current is decreasing.

• the current reaches zero when the diode turns off. The first positive value of ωt at which the current is zero is called the "extinction angle" and denoted by β.

• thus when $\omega t = \beta$ the current is zero

$$i(\beta) = \frac{V_m}{Z}\left[sin(\beta - \theta) + sin(\theta) e^{-\frac{\omega t}{\omega\tau}} \right] = 0$$

Example 2

Consider the half-wave rectifier with RL load shown in Figure (7a). The circuit has $R = 100\ \Omega$, $L = 0.1\ H$, $\omega = 2\pi f = 377\ rad / s$, and $V_m = 100\ V$.

Determine

(a) an *expression* for the *load current*.

(b) the *average* value of the current.

(c) the *rms* value of the current.

(d) the *power* absorbed by the load.

(e) the *power factor*.

Solution

From the given parameters

$$\omega L = 37.7\ \Omega$$

$$z = \sqrt{\left(R^2 + (\omega L)^2\right)} = \sqrt{100^2 + 37.7^2} = 106.9\ \Omega$$

$$\theta = tan^{-1}\left(\frac{\omega L}{R}\right) = 20.7^o = 0.361\ rad$$

$$\alpha = sin^{-1}(0) = 0, \pm\pi, \pm 2\pi$$

$$\tau = \frac{L}{R} = 1\ ms$$

(a) Using the current relationship gives

$$i(\omega t) = \frac{V_m}{Z} sin(\omega t - \theta) + \frac{V_m}{Z} sin\,\theta\ e^{-\omega t/\omega\tau}$$

or

$$i(\omega t) = 0.936\ sin(\omega t - 0.361) + 0.331 e^{-\omega t/0.377}\ A$$

This relationship is valid for $0 \le \omega t \le \beta$ where β is found from

$$0 = \sin(\beta - 0.361) + \sin(0.361)e^{-\beta/0.377}$$

Solving this equation by a numerical method gives

$$\beta = 3.5 \ rad \ = 201^{o}$$

(b) The *average current* is determined from the load current as

$$I_{dc} = \frac{1}{2\pi}\int_{0}^{\beta} i(\omega t)d(\omega t)$$

or

$$I_{dc} = \frac{1}{2\pi}\int_{0}^{3.5}\left[0.936 \sin(\omega t - 0.361) + 0.331e^{-\frac{\omega t}{0.377}}\right]d(\omega t)$$

Using a numerical integration method gives

$$I_{dc} = 0.308 \ A$$

(c) The *rms* current is

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{3.5}\left[0.936 \sin(\omega t - 0.361) + 0.331e^{-\frac{\omega t}{0.377}}\right]^{2} d(\omega t)}$$

or

$$I_{rms} = 0.474 \ A$$

(d) The power absorbed by the resistor is

$$I_{rms}^{2} R = (0.474)^{2} \, 100 = 22.4 \ W.$$

The power absorbed by the conductor is zero

(e) The power factor is computed from

$$pf = \frac{p}{S} = \frac{p}{V_{s,rms} I_{rms}} = \frac{22.4}{\left(\frac{100}{\sqrt{2}}\right)(0.474)} = 0.67$$

DC Converters (Choppers)

A *dc chopper* converts directly from *dc to dc* and is also known as a *dc-to-dc converter*. A chopper can be considered as a *dc* equivalent to an *ac transformer* with continuously variable turns ratio. Like a transformer it can be used to *step-down* or *step-up* a *dc* voltage source.

Principle of Step-Down Operation

Consider Figure (8a). When switch is closed for a time t_1, the input voltage source V_s appears across the load. If the switch is turned and remains off for time t_2, the voltage across the load is zero. In practice a finite voltage drop across the chopper ranging from 0.5 to 2 V is experienced, however for simplicity we neglect the voltage drop in the following analysis. The waveforms of the output voltage and load current are shown in Figure (8b).

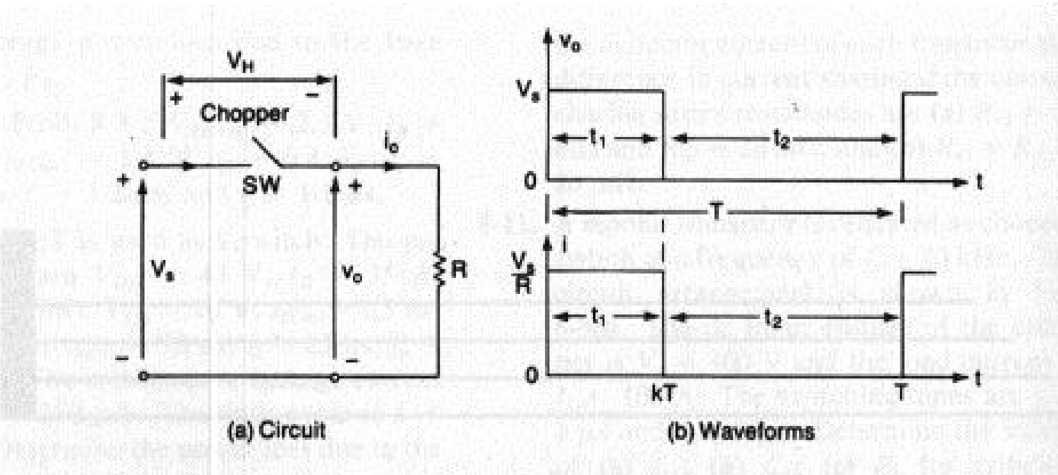

Figure (8)

The average output voltage is

$$V_{output,dc} = \frac{1}{T}\int_0^{t_1} v_o dt = \frac{t_1}{T} V_s = f\, t_1 V_s = kV_s$$

The average load current is

$$I_{dc} = \frac{V_{dc}}{R} = k\frac{V_s}{R}$$

The parameter $k = \frac{t_1}{T}$ is called the duty cycle of the chopper, and *f* is the chopping frequency.

The rms value of the output voltage is

$$V_{output,rms} = \sqrt{\frac{1}{T}\int_0^{kT} v_o^2\, dt} = \sqrt{k}\; V_s$$

Assuming a lossless chopper, the input power to the chopper is the same as the output power and is determined from

$$P_{input} = P_{output} = \frac{1}{T}\int_0^{kT} v_o \, i \, dt = k\frac{V_s^2}{R}$$

The *effective* input resistance seen by the source is

$$R_{input} = \frac{V_s}{I_{dc}} = \frac{R}{k}$$

Remarks

• The duty cycle *k* can be varied from 0 to 1 by varying t_1, *T* or *f*. Therefore the output voltage can be varied from 0 to V_s by controlling *k*. As a consequence, the power flow into the circuit cab be controlled.

• When the chopping frequency is kept constant and the duty cycle is varied, the chopper is said to be in *constant frequency operation* . In this mode of operation the width of the pulse is varied and this is referred to as *pulse-width-modulation* (PWM).

• When the chopping frequency is varied and the duty cycle is kept constant, the chopper is said to be in *variable frequency operation*. In this mode of operation the width of the frequency is varied over a wide range to obtain the full output voltage range. This is not a practical option as it generates harmonics at unpredictable frequencies.

Example

Consider the dc-dc converter shown in Figure (8). The dc converter has a resistive load of $R = 10\ \Omega$ and the input voltage is $V_s = 220\ V$. When the converter switch remains on, its voltage drop is $v_{ch} = 2\ V$ and the frequency is $f = 1\ kHz$. If the duty cycle is *k=50*%, determine:

(a) the average output voltage $V_{o,a}$.
(b) the rms output voltage $V_{o,rms}$.
(c) the converter efficiency.
(d) the effective input resistance of the converter R_i.

Solution

Given: $V_s = 220V;\ k = 0.5;\ R = 10\Omega;\ v_{ch} = 2V$

(a) The average output voltage is

$$V_{dc} = \frac{1}{T}\int_0^{t_1} v_o dt = \frac{t_1}{T}(V_s - v_{ch}) = ft_1(V_s - v_{ch}) = k(V_s - v_{ch})$$

$= 0.5(220\text{-}2) = 109V.$

(b) The *rms* output voltage is

$$V_o = \sqrt{\left(\frac{1}{T}\int_0^{kT} v_o^2 dt\right)} = \sqrt{k}(V_s - v_{ch}) = \sqrt{0.5}(220 - 2) = 154.15\ V$$

(c) The efficiency is

$$\xi = \frac{p_o}{p_i}$$

The output power is found from

$$p_o = \frac{1}{T}\int_0^{kT} \frac{v_o^2}{R} dt = \frac{1}{T}\int_0^{kT} \frac{(V_s - v_{ch})^2}{R} dt = k\frac{(V_s - v_{ch})^2}{R}$$

$$= 0.5x\frac{(220 - 2)^2}{10} = 2376.2\ W$$

The input power to the converter is found as

$$p_i = \frac{1}{T}\int_0^{kT} V_s i dt = \frac{1}{T}\int_0^{kT} \frac{V_s(V_s - v_{ch})}{R} dt = k\frac{V_s(V_s - v_{ch})^2}{R}$$

$$= 0.5x220x\frac{(220 - 2)}{10} = 2398\ W$$

Thus the converter efficiency is

$$\xi = \frac{p_o}{p_i} = \frac{2376.2}{2398} = 99.09\%$$

(d) The effective input resistance of the converter is

$$R_i = \frac{V_s}{I_a} = \frac{V_s}{kV_s / R} = \frac{R}{k} = \frac{10}{0.5} = 20\ \Omega$$