The Transformer

Transformers consist of two or more windings on a core made of magnetic material with high permeability. The windings are linked by a common magnetic field through the core. In power distribution systems, transformers provide a convenient means for obtaining the high voltages necessary for the economical transmission of electrical energy over long distances. Power system transformers operating at sinusoidal 60 Hz voltages are designed on magnetic cores of laminated sheet steel. In power electronics applications, transformers are usually operated at frequencies from several tens of kHz to several MHz and are built on ferrite magnetic cores that have much smaller losses at high frequencies. In both power systems and power electronics applications, the transformer operating principles are the same and the circuit models suitable for system analyses are similar. The purpose of this lab assignment is to examine transformer operation, develop a transformer circuit model, and show how the model can be used in applications. In the lab, experiments are performed on a 60 Hz power transformer but the modeling and measurement techniques will also apply to high-frequency transformers in power electronics applications.

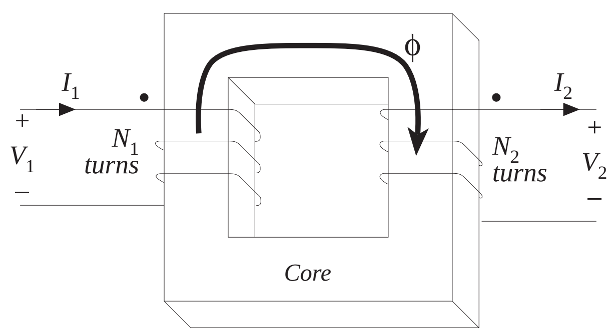

Fig. 1 Two-winding transformer.

1. The Ideal Transformer

The simplest transformer consists of two windings of insulated copper wire wound around a core of magnetic material with very high relative permeability μ_r, as shown in Fig.1. One winding, having N_1 turns of wire, is designated the primary winding, and the other, having N_2 turns, the secondary. An AC voltage V_1 applied to the primary results in AC currents I_1 and I_2, and an AC flux ϕ through the core. Assuming that all losses can be neglected, and that the flux ϕ exists only in the core, i.e., that the flux through the primary and the secondary windings is exactly the same, then Faraday's law for the voltages induced in the windings leads to:

$$V_1 = N_1 \frac{d\phi}{dt}$$
$$V_2 = N_2 \frac{d\phi}{dt} \qquad (1)$$

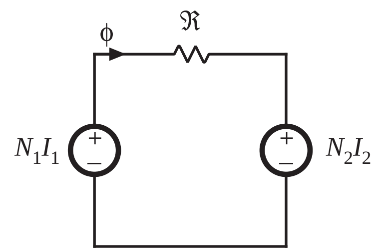

Fig. 2 Magnetic circuit model of the transformer of Fig. 1.

We can now eliminate $d\phi/dt$ from this equation, to show that the winding voltages are related through the number of turns:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (2)$$

Figure 2 contains the magnetic circuit model of the transformer. The flux ϕ in the core can be found by solving this model, as follows:

$$\phi\Re = N_1 I_1 - N_2 I_2 \qquad (3)$$

Here, $\Re$ is the equivalent reluctance of the magnetic path through the core. If one assumes that the magnetic core is made of material with very high permeability, then the reluctance is very small: $\Re \to 0$. This implies that $N_1I_1 - N_2I_2 \approx 0$, and so the terminal currents are also related through the turns ratio:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} \qquad (4)$$

Equations (2) and (4) imply that $V_1I_1 = V_2I_2$, i.e., that the instantaneous input power to the transformer primary is equal to the instantaneous output power delivered by the transformer secondary. These equations define what is called the ideal transformer. The ideal transformer is a lossless device in which the terminal voltages and currents (defined according to the reference polarities of Fig.1) satisfy the relationships (2) and (4).

2. The Physical Transformer

A practical transformer is not ideal in its behavior. In the conversion process between primary and secondary, energy is lost in the form of heat. The resistance of the winding leads to I^2R *copper losses*. These losses can be modeled by inserting resistances in series with each winding. *Core loss*, or *iron loss*, arises in the core material; two mechanisms lead to this loss. First, the hysteresis of the *B-H* magnetizing characteristic implies that not all of the magnetic energy stored within the core can be recovered. Second, ac variation of the magnetic field causes eddy currents to flow within the core material itself; these induced currents lead to I^2R losses in the core material. Core loss is typically approximately proportional to the square of the ac flux density in the core, and is usually modeled using a shunt resistor (i.e., a resistor in parallel with one of the transformer windings).

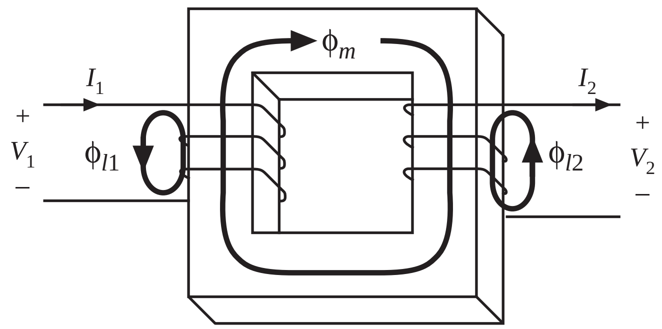

Fig. 3. Typical flux paths in a physical transformer: mutual flux ϕ_m, and leakage fluxes ϕ_{l1} and ϕ_{l2}.

Physical transformers also differ from the ideal transformer in ways that do not involve power loss. Since the transformer is constructed using coils of wire wound on a magnetic core, one would expect the windings to exhibit inductance. The nonzero reluctance $\Re$ of the core causes the windings to exhibit an effective shunt inductance L_m, called the *magnetizing inductance*. This inductance models magnetization of the core. If we disconnect the secondary winding, then we are left with the primary winding on the magnetic core—an inductor, having a value approximately equal to L_m.

The flux paths of a typical physical inductor are illustrated in Fig. 3. Because of the nonzero reluctance of the core material, some flux passes through the air and does not link both windings. This is called *leakage flux*, illustrated in Fig. 3 by the fluxes ϕ_{l1} and ϕ_{l2}. The effects of these fluxes can be modeled by adding inductances in series with the to the ideal transformer windings.

An equivalent circuit that models all of the physical processes described above is given in Fig. 4(a). Impedances Z_1 and Z_2 model the magnetic leakage and the winding losses of the primary and the secondary, respectively. As described above, shunt inductor L_m is the magnetizing inductance, and R_m models the core loss. In a practical transformer, impedances Z_1 and Z_2 are much smaller than $Z_m = j\omega L_m \parallel R_m$. The transformer circuit model of Fig. 4(a) is called the *T-model* because the impedances Z_1, Z_2, Z_m, that are added to the ideal transformer form a T.

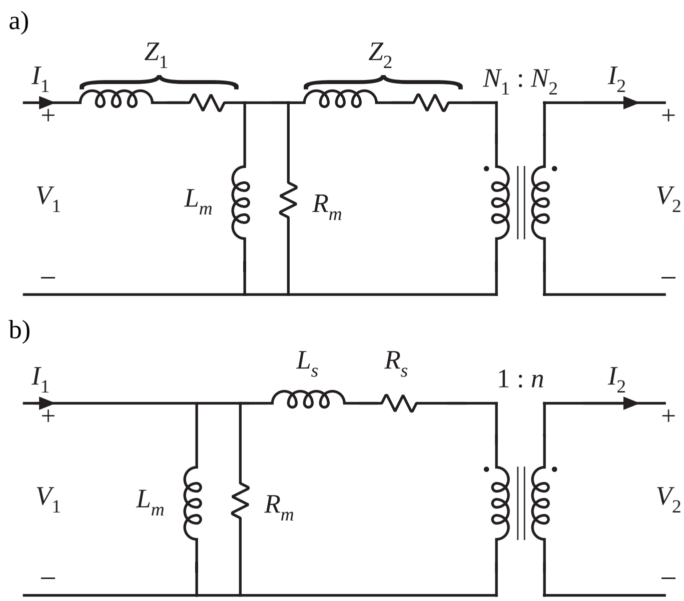

Fig. 4. Equivalent circuit models of the physical transformer: (a) T-model, (b) cantilever model.

The advantage of the T-model is that it is based on physical reasoning, and hence each element has a simple physical interpretation. However, solution of the T-model can lead to complex algebra. It is possible to reduce the T-model to an equivalent form that is more convenient for circuit analysis; the result is shown in Fig. 4(b) and is known as the *cantilever model*. Since Z_1 and Z_2 are typically much smaller than Z_m, then Z_s is approximately equal to ($Z_1 + Z_2$). The effective turns ratio n of the ideal transformer in the cantilever model is equal to the voltage ratio V_2/V_1 of the physical transformer; with small series impedances, this is approximately the same as the turns ratio of the physical windings: $n \approx N_2/N_1$. The cantilever model has fewer parameters than the T-model, and its main advantage is that the parameters can be obtained directly from short- and open-circuit tests as discussed in the next section. One should note that the transformer, because of the highly nonlinear characteristic of the magnetic core material, is a non-linear device, and the equivalent circuit "constants" are valid for only one operating condition. Measurements of the parameters of the transformer equivalent circuit are usually done at the nominal operating conditions. Nevertheless, the model with parameters measured under nominal conditions is approximately valid and useful for analyses over a range of operating conditions.

3. Open- and Short-Circuit Tests

In this section, we discuss the measurements that can be used to determine the parameters of the cantilever model of Fig. 4(b). Two measurement setups are used: the open-circuit test to determine the parameters L_m, R_m, and n, and the short-circuit test to determine L_s and R_s.

The open-circuit test is usually performed by energizing the primary winding at rated frequency and rated voltage with the secondary open circuited ($I_2 = 0$). The measurement setup is shown in Fig. 5(a). Since $I_2 = 0$, the open-circuit test gives

$$\frac{V_1}{I_1} = Z_m \tag{5}$$

where V_1 and I_1 are phasors that represent the primary voltage and current, and Z_m is the complex impedance $Z_m = j\omega L_m || R_m$. Using a wattmeter, we can measure the (real) power P lost in the transformer in the open-circuit test. Since R_m is the only lossy (resistive) element in the equivalent circuit, the resistance R_m can be found as:

$$R_m = \frac{\| V_1 \|^2}{P} \tag{6}$$

where $\| V_1 \|$ is the RMS voltage applied to the primary. Inductance L_m can then be determined from:

$$\frac{1}{\| Z_m \|} = \frac{\| I_1 \|}{\| V_1 \|} = \sqrt{\left(\frac{1}{\omega L_m}\right)^2 + \left(\frac{1}{R_m}\right)^2} \tag{7}$$

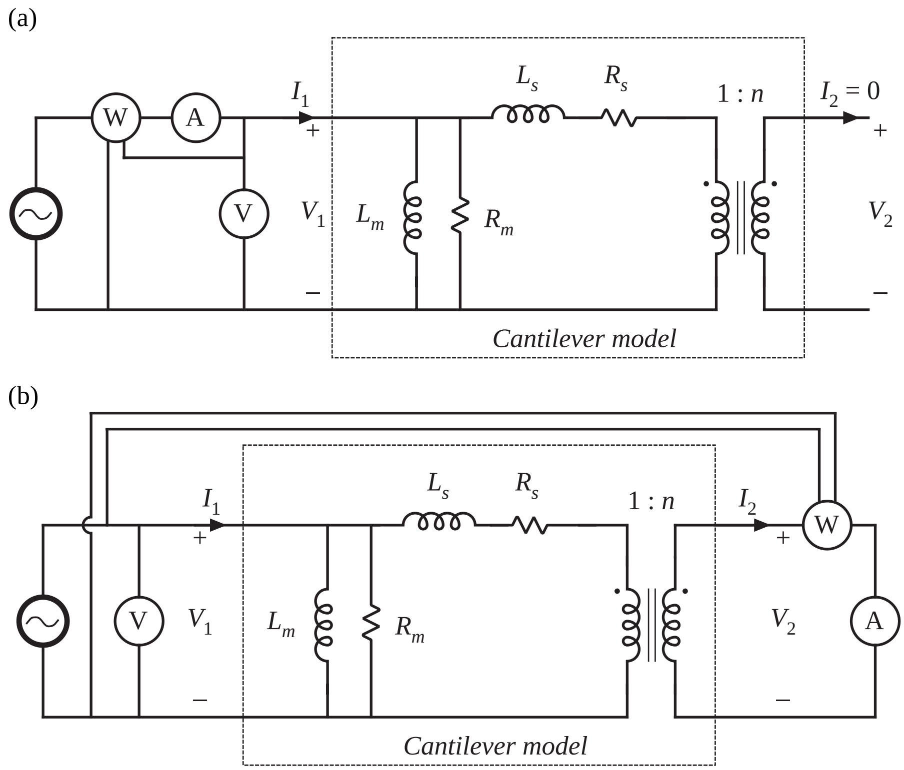

Fig. 5. Measurement of parameters in the cantilever model: (a) open-circuit test, (b) short-circuit test.

In our laboratory experiment, the core losses will be neglected, i.e., we will assume that $R_m \rightarrow \infty$. In this case, L_m can be found directly from (7):

$$L_m = \frac{1}{\omega} \frac{\| V_1 \|}{\| I_1 \|} \tag{8}$$

The effective turns ratio also follows directly from the open-circuit test:

$$n = \frac{\| V_2 \|}{\| V_1 \|} \tag{9}$$

The short-circuit test is usually performed by short circuiting the secondary (V_2 = 0), and applying a (reduced) voltage to the primary at rated frequency and of such magnitude to cause rated current to flow. The measurement setup is shown in Fig. 5(b). By measuring the primary voltage V_1 and the short-circuited secondary current I_2, we can determine the values of R_s and L_s. Note that $V_2 = 0$, so that

$$\frac{V_1}{Z_s} = nI_2 \tag{10}$$

$$Z_s = \frac{1}{n} \frac{V_1}{I_2} \tag{11}$$

$$L_s = \frac{1}{n\omega} \frac{\| V_1 \|}{\| I_2 \|} \sin(\theta) \tag{12}$$

$$R_s = \frac{1}{n} \frac{\| V_1 \|}{\| I_2 \|} \cos(\theta) \tag{13}$$

where θ is the phase angle between the voltage V_1 and the current I_2. This measurement requires finding the RMS values of V_1 and I_2, as well as the phase difference between the excitation V_1 and the short-circuited secondary current I_2. If a wattmeter is available and connected as shown in Fig. 5(b), then the resistance R_s can be found from the measured power loss:

$$R_s = \frac{P}{n \| I_2 \|^2} \tag{14}$$

Once R_s is determined, L_s can be found from (11):

$$L_s = \frac{1}{\omega} \sqrt{\| Z_s \|^2 - R_s^2} \tag{15}$$

An alternate method for the short-circuit test is to apply rated current to the secondary, with the primary short-circuited. The equations for the model series parameters are similar.

4. Transformer Nameplate Ratings

On the nameplate of a utility power transformer, one typically sees a rating such as: 480V : 120V, 5kVA. The specified voltages are the rated winding voltages, measured under full load conditions. The actual turns ratio is not exactly equal to the ratio of the rated voltages, because of the voltage drops in the series impedances of the transformer model.

The magnitude of the total series impedance in the cantilever model, $\| Z_s \|$, is typically specified in the per-unit system as a percentage such as "3%". This means that the series impedance magnitude, referred to a given winding, is equal to

$$\| Z_s \| = \left(\frac{\text{specified percentage}}{100} \right) \frac{(\text{rated VA})}{(\text{winding rated voltage})^2} \tag{16}$$

So, for example, the value of $\| Z_s \|$ of a transformer rated 480V : 120V, 5kVA, 3%, is $(0.03)(5000)/(120)^2 = 0.0104\Omega$ referred to the 120V side.

5. The Autotransformer

A transformer can also be connected as an autotransformer to achieve different voltage ratios. The primary and secondary are no longer isolated. An example is illustrated in Fig. 6. In the autotransformer connection, one winding is connected between the input and output. In consequence, the transformer produces the difference between the input and output voltages.

When the input and output voltages are similar in magnitude, the rated VA of the transformer windings can be much smaller than the output power. Hence, the size of the autotransformer can be relatively small.

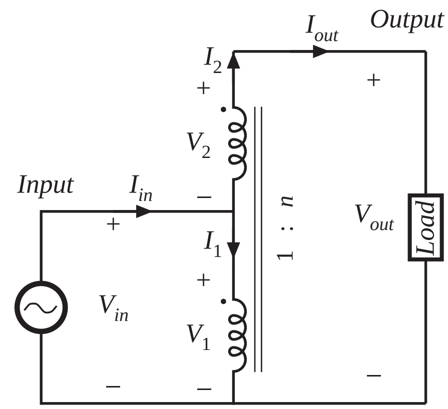

Fig. 6. A common approach for connection of a two-winding transformer as an autotransformer.

PROBLEMS

1. Solve Eq. (7) for L_m

2. Derive Eq. (14).

3. A two-winding transformer is rated 480 V : 120 V, 1 kVA. The results of open- and short-circuit tests are as follows:

0.15A, 120V, 12W

2.083A, 12V, 18W

(a) Which is the short-circuit test? Was it performed on the 480 V winding, or on the 120 V winding?

(b) The transformer is now operated under rated conditions at unity power factor. Determine: (1) the copper loss, (2) the core loss, and (3) the efficiency.

4. A 60 Hz transformer that one might find mounted on the telephone pole in a Boulder back yard is rated 13600 V : 240 V, 40 kVA. Its cantilever-model parameters are (referred to the 13.6kV winding): L_m = 250 H, R_m = 200kΩ, R_s = 35Ω, L_s = 0.3H, n = 0.01777

(a) Find the power loss in the transformer during open-circuit test.

(b) Find the power loss in the transformer during short-circuit test.

(c) The secondary is connected to a resistive load, having power 0 W < P_{load} < 40 kW. Find an expression for $\| V_2 \|$ as a function of P_{load} and plot it. Find how much the load voltage varies over the specified range of load powers.

5. A French engineer living in England decides to construct a 50 Hz autotransformer so that he may operate his French 220 V appliances from the English 240 V service.

(a) There are two ways to connect a two-winding autotransformer as a step-down autotransformer. Sketch the best way (whose rated VA is lowest).

(b) For the method of part (a), determine the winding voltage and current ratings and the transformer VA ratings, to supply a 220 V, 2.2 kVA load from a 240 V input.