Three-Phase Transformer Connections

Ideal transformers can be employed in three-phase systems to shift the phases of voltages and currents, change their waveshapes, and convert three-phase ac to two-phase, five-phase, six-phase, or other ac systems. A large number of circuits are possible, even when it is desired simply to change the voltage magnitude. The more sophisticated transformer connections find significant application in modern high-power rectifier circuits, where they lead to improved ac line current waveforms.

The purpose of this experiment is to become familiar with voltage and current phasors in three-phase systems, to learn how ideal (or near-ideal) transformers can be used to shift phase in three-phase systems, and to explore some basic transformer connections.

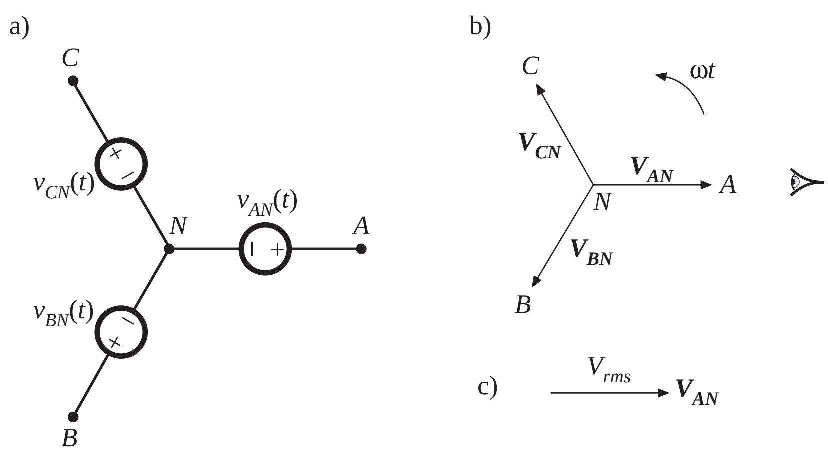


Fig. 1 Three-phase source: (a) schematic, (b) phasor diagram, (c) the phasor $\boldsymbol{V_{AN}}$.

1. Review of Basic Three-Phase Systems

Figure 1 illustrates the circuit and phasor diagram of a basic three-phase ac source. The circuit of Fig. 1(a) contains a neutral point N that is normally connected to Earth ground. The three phases (or lines) are labeled A, B, and C. The voltage sources produce sinusoids having rms values V_{rms}, which are shifted in phase by 120˚ and 240˚. These voltages can be expressed in the time domain as follows:

$$v_{AN}(t) = \sqrt{2}\ V_{rms} \cos\left(\omega t\right)$$
$$v_{BN}(t) = \sqrt{2}\ V_{rms} \cos\left(\omega t - 120°\right)$$
$$v_{CN}(t) = \sqrt{2}\ V_{rms} \cos\left(\omega t - 240°\right) \tag{1}$$

In the power area, phasor magnitudes are labeled using rms values as in Fig. 1(c). These voltages can be represented by the following phasors:

$$\boldsymbol{V_{AN}} = V_{rms} \angle 0°$$
$$\boldsymbol{V_{BN}} = V_{rms} \angle{-120°}$$
$$\boldsymbol{V_{CN}} = V_{rms} \angle{-240°} \tag{2}$$

Figure 1 and Eq. (1) also illustrate the naming convention commonly used for the voltages of three-phase systems. Voltages are designated using two subscripts. The voltage at the point designated by the first subscript is measured with respect to the point denoted by the second subscript. Hence, $\boldsymbol{V_{AN}}$ is the voltage from line A to the neutral point; the phasor representing $\boldsymbol{V_{AN}}$ is drawn with the arrowhead at A and the tail at N. Note that $\boldsymbol{V_{AN}} = -\boldsymbol{V_{NA}}$.

Any signal can be arbitrarily chosen as the zero-phase reference; the phases of other signals are then represented with respect to this zero phase reference. In Fig. 1(b) and Eq. (1), the line-to-neutral voltage $\boldsymbol{V_{AN}}$ is chosen as the zero phase reference. As time increases, the phasors rotate in the counterclockwise direction, with speed ω. An observer standing to the right of the phasor diagram would see phasor $\boldsymbol{V_{AN}}$ rotate by, followed by phasor $\boldsymbol{V_{BN}}$, followed by phasor $\boldsymbol{V_{CN}}$. This is called *ABC* phase sequence. The opposite (*CBA*) phase sequence is obtained when the phasors rotate in the clockwise direction or when two of the phases (such as B and C) are interchanged. Throughout the remainder of this discussion, *ABC* phase sequence is assumed.

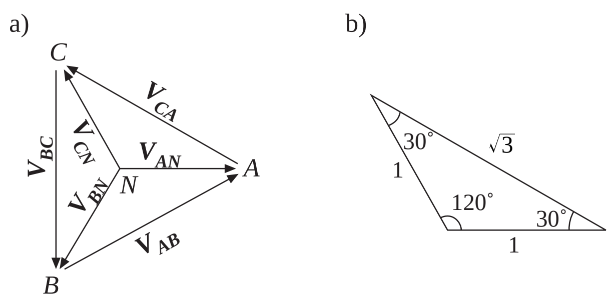


Fig. 2 Three-phase system phasors: (a) construction of line-to-line voltages, (b) geometry of line-to-line vs. line-to-neutral voltage phasors.

Figure 2 illustrates construction of the line-to-line voltages. The phasors for $\boldsymbol{V_{AB}}$, $\boldsymbol{V_{BC}}$, and $\boldsymbol{V_{CA}}$ are given by

$$\boldsymbol{V_{AB}} = \boldsymbol{V_{AN}} - \boldsymbol{V_{BN}} = \boldsymbol{V_{AN}} + \boldsymbol{V_{NB}} = V_{rms} \angle 0° + V_{rms} \angle 60° = \sqrt{3}\ V_{rms} \angle 30°$$
$$\boldsymbol{V_{BC}} = \sqrt{3}\ V_{rms} \angle{-90°}$$
$$\boldsymbol{V_{CA}} = \sqrt{3}\ V_{rms} \angle{-210°} \tag{3}$$

Figure 2(b) illustrates the relationship between the line-to-neutral and the line-to-line voltages. In general, the line-to-line voltages are shifted in phase by 30˚, and are increased in magnitude by a factor of $\sqrt{3}$, with respect to the line-to-neutral voltages.

It is customary to specify the rms line-to-line voltage of three-phase systems. For example, the 240 V three-phase system in our power lab has an rms line-to-line voltage of 240 V. The line-to-neutral voltage of this system would be $240/\sqrt{3} = 138.6 \text{ V}$.

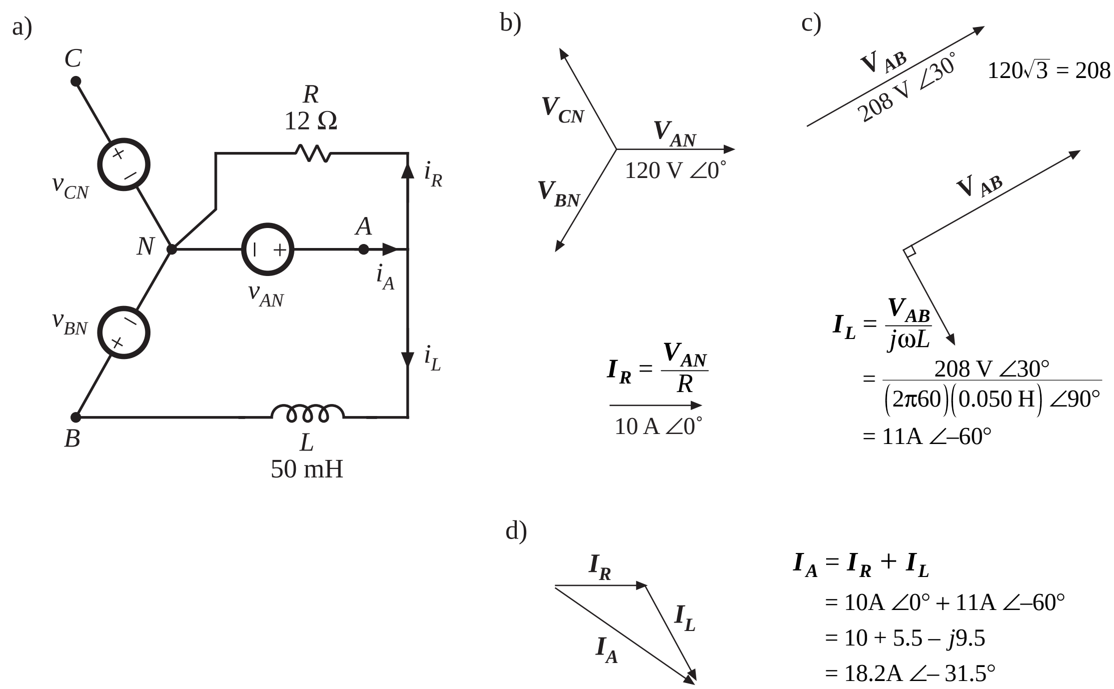


Fig. 3 Current phasor example.

Construction of current phasors

An example that illustrates construction of current phasors is given in Fig. 3. A resistor is connected between line A and neutral N, while an inductor is connected between lines A and B. It is desired to construct the phasors representing the currents in the resistor, inductor, and the phase A line current. The 60 Hz voltages are assumed to be as in Figs. 1 and 2, with rms voltage V_{rms} = 120 V. The line-to-line voltages have magnitude The current $\boldsymbol{I_R}$ through the 12 Ω resistor is given by Ohm's law:

$$\boldsymbol{I_R} = \frac{\boldsymbol{V_{AN}}}{R} = \frac{120 \angle 0°}{12} = 10\angle 0° \tag{4}$$

The current through the resistor is in phase with the applied voltage, the line-to-neutral voltage $\boldsymbol{V_{AN}}$. The phasor $\boldsymbol{I_R}$ is illustrated in Fig. 3(b).

The inductor L is connected line-to-line between phases A and B. The inductor current is therefore given by

$$\boldsymbol{I_L} = \frac{\boldsymbol{V_{AB}}}{j\omega L} = \frac{208\ \angle 30°}{(2\pi 60)(0.050)\ \angle 90°} = 11\angle{-60°} \tag{5}$$

The line-to-line voltage $\boldsymbol{V_{AB}}$ leads the line-to-neutral voltage $\boldsymbol{V_{AN}}$ by 30˚. The inductor current lags this applied voltage by 90˚. Hence, the phase of $\boldsymbol{I_L}$ is –60˚. Construction of $\boldsymbol{I_L}$ is illustrated in Fig. 3(c).

The phase A line current $\boldsymbol{I_A}$ is determined by the node equation

$$\boldsymbol{I_A} = \boldsymbol{I_R} + \boldsymbol{I_L} \tag{6}$$

Addition of these phasors leads to

$$\begin{aligned} \boldsymbol{I_A} = \boldsymbol{I_R} + \boldsymbol{I_L} &= 10\angle 0° + 11\angle{-60°} \\ &= 10 + 5.5 - j9.5 \\ &= 18.2\angle{-31.5°} \end{aligned} \tag{7}$$

Construction of $\boldsymbol{I_A}$ is illustrated in Fig. 3(d).

Balanced wye- and delta-connected loads

Figure 4 illustrated several balanced three-phase loads. In Fig. 4(a), resistors of value R are connected in a *wye* configuration, from line to neutral. The line currents are in phase with the respective line-to-neutral voltages. The neutral points of the source and load may be connected, or this connection may be omitted. Figure 4(b) illustrates resistors of value $3R$, connected in a *delta* configuration, from line to line. It can be shown that the magnitudes and phases of the line currents are identical to the respective line currents of Fig. 4(a). In general, a balanced resistive load leads to line currents that are in phase with the respective line-to-neutral voltages, regardless of how the load resistors are actually connected.

In Fig. 4(c), inductors of value L are wye-connected, to form a balanced inductive load. The line currents now lag the respective line-to-neutral voltages by 90˚. The delta-connected inductors of Fig. 4(d), having value $3L$, lead to identical line currents.

In general, it can be shown that balanced three-phase systems with sinusoidal waveforms exhibit the following properties:

1. The instantaneous three-phase power $v_{AN}(t)i_A(t) + v_{BN}(t)i_B(t) + v_{CN}(t)i_C(t)$ is constant, and is equal to three times the average power of one phase.

2. No current flows in a wire connecting the neutral points of wye-connected sources and loads.

3. The total three-phase reactive power is equal to three times the reactive power of one phase.

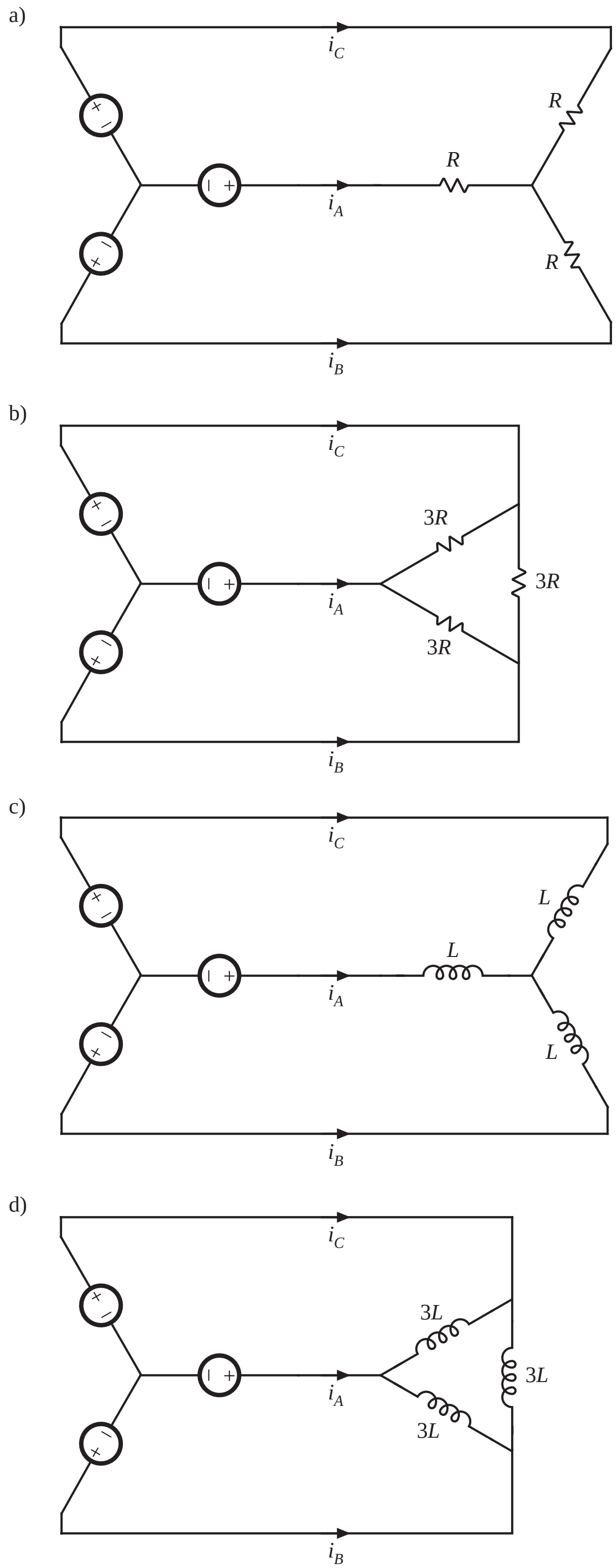


Fig. 4 Several examples of balanced three-phase loads: (a) wye-connected resistive load, (b) delta-connected resistive load, (c) wye-connected inductive load, (d) delta-connected inductive load.

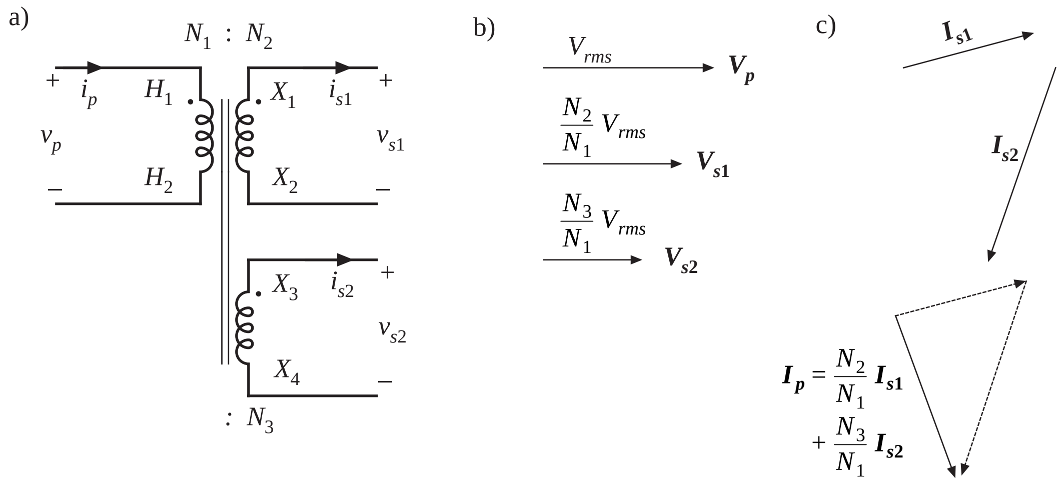


Fig. 5 Ideal three-winding transformer example. (a) Schematic. (b) Winding voltage phasors must point in the same direction. Their lengths scale by the turns ratios. (c) Primary current phasor is the vector sum of the reflected secondary currents.

2. Phasor Relationships in the Ideal Transformer

An ideal transformer containing a primary winding H_1-H_2 and two secondary windings X_1-X_2, X_3-X_4 is illustrated in Fig. 5(a). In phasor form, the terminal equations of the transformer are:

$$\begin{aligned} \boldsymbol{V_{s1}} &= \frac{N_2}{N_1}\boldsymbol{V_p} \\ \boldsymbol{V_{s2}} &= \frac{N_3}{N_1}\boldsymbol{V_p} \\ \boldsymbol{I_p} &= \frac{N_2}{N_1}\boldsymbol{I_{s1}} + \frac{N_3}{N_1}\boldsymbol{I_{s2}} \end{aligned} \tag{8}$$

Here, the defined polarity of each winding voltage is taken to be positive at the dot. The defined directions of positive winding currents is taken to be into the dot on the primary winding and out of the dots on the secondary windings.

Since the winding voltages are proportional, they are all in phase. Thus, the phasors representing the secondary voltages are in phase with the primary voltage phasor. The magnitudes of the secondary phasors are scaled according to the turns ratios. Typical winding voltage phasors are illustrated in Fig. 5(b).

The primary winding current is equal to the sum of the reflected secondary currents. Hence, to construct the phasor that represents the primary current, one must first scale the magnitudes of the secondary current phasors according to their respective turns ratios, and then add the result. Figure 5(c) illustrates typical winding current phasors. The secondary winding current phasors are determined by the external secondary loads. The primary winding current phasor is constructed by vector addition, as described above.

3. Construction of Voltage Phasors in Three-Phase Transformer Circuits

In principle, the above rules are sufficient to analyze any three-phase circuit. However, some three-phase transformer circuits are quite complex, and hence in practice it becomes necessary to employ extra care and effort in constructing the phasor diagrams. The procedure below is an approach that aids in keeping track of the correct phasor directions and polarities:

Given phasors of 3ø input to circuit

Step 1. Construct phasors that represent the primary winding voltage polarities.

Step 2. Sketch arrows on the circuit diagram primary windings, with phasor arrowheads and tails oriented next to windings in manner consistent with step 1. Label each transformer with its name, and label the transformer names on the phasor diagram next to the corresponding primary voltage phasors.

Step 3. Sketch arrows on circuit diagram secondary windings, with arrowheads oriented consistently with primary phasors (step 2) and winding polarity marks. Sketch the phasors representing the secondary voltages: in phase with respective primary phasors, and with magnitudes scaled by turns ratios.

Step 4. Follow secondary circuit diagram to see how the secondary phasors connect (head-to-head, head-to-tail, etc.). Construct secondary phasor diagram accordingly.

Figure 6(a) illustrates a wye-delta connected three-phase transformer circuit example, using 2:1 turns ratios. Let us analyze this circuit to determine the magnitudes and phases of the secondary voltages, with respect to the corresponding primary voltages.

The given input three-phase voltage is the European 380 V. The input line-to-neutral voltage is therefore $380/\sqrt{3} = 220$ V .

All transformer windings have been clearly labeled. The 220 V line-to-neutral input voltage phasors are shown in Fig. 6(b). Since the transformer primary windings are connected in wye, the primary voltage phasors coincide with the line-to-neutral input voltage phasors. Figure 6(b) illustrates labeling these voltage phasors, with the corresponding transformer names T_1, T_2, and T_3. The H_1 terminals are connected to the ac

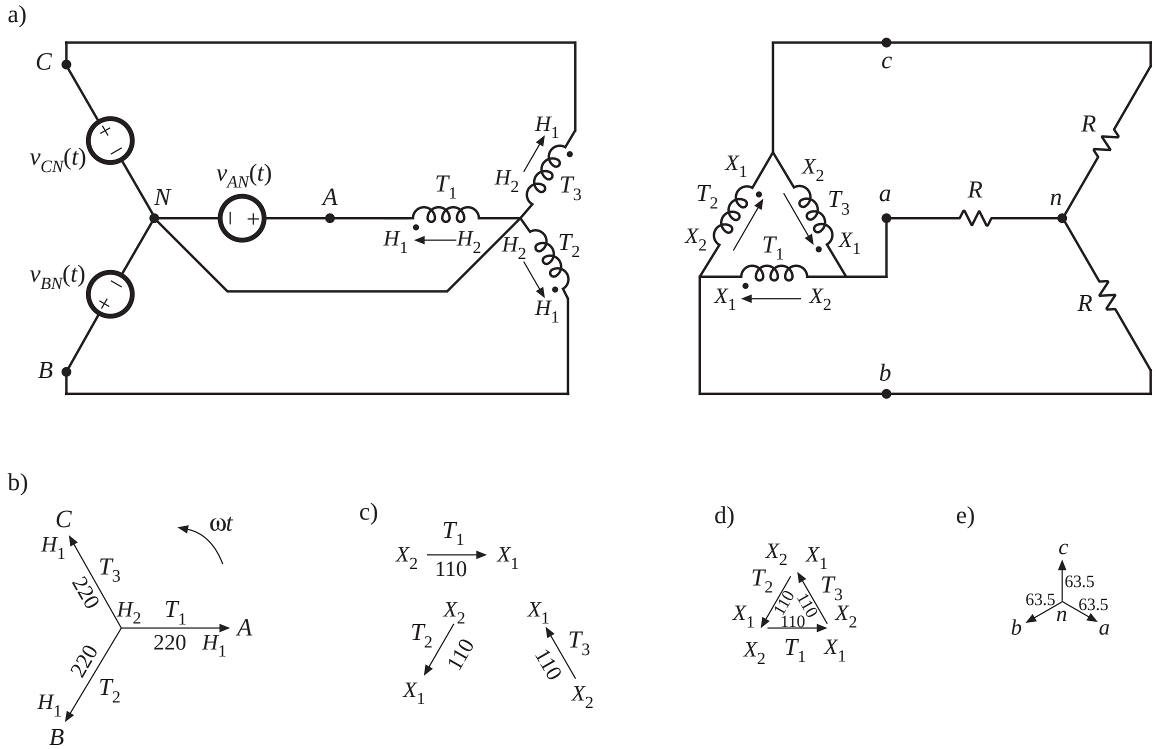


Fig. 6 Wye-delta transformer connection: (a) schematic, (b) primary winding voltage phasors, (c) secondary winding voltage phasors, (d) delta connection of secondary voltage phasors, (e) wye-connected load voltage phasors.

lines, while the H_2 terminals are connected to the neutral point. On the schematic diagram of Fig. 6(a), arrows are drawn next to the transformer primaries that indicate the polarities of the primary voltage phasors: the arrowhead of the T_1 primary voltage phasor is at the H_1 terminal, while the tail is at the H_2 terminal.

Since the primary winding voltage phasors have arrowheads at the H_1 (dot) terminals, the secondary winding voltage phasors must have arrowheads at the X_1 (dot) terminals. The transformer 2:1 turns ratios cause the phasors representing the secondary winding voltages to have magnitude 220/2 = 110 V. These phasors are parallel to the respective primary winding voltage phasors, as illustrated in Fig. 6(c).

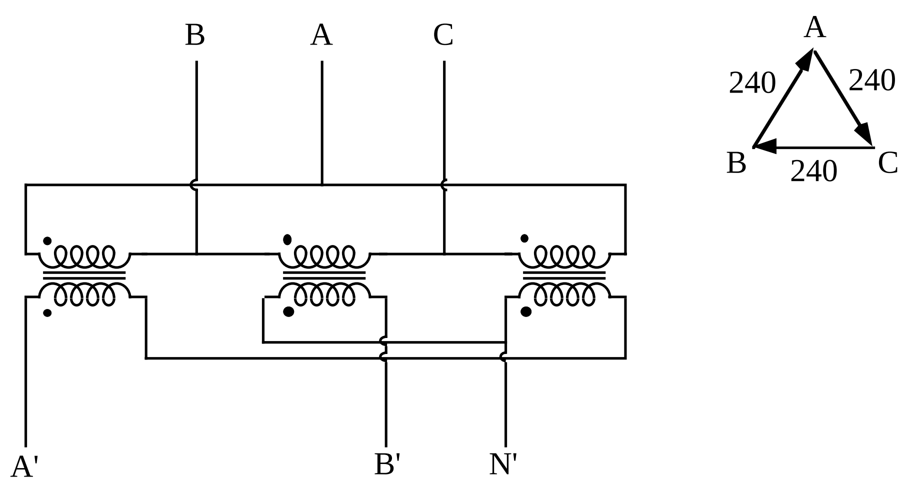
We can now construct the output-side voltage phasors. The secondary winding voltage phasors are placed together according to the circuit diagram. The arrowhead of the T_1 secondary winding phasor (X_1) is connected to output phase a. Again, this phasor is drawn parallel to the T_1 primary winding voltage phasor. The tail of the T_1 secondary phasor (X_2) is connected to output phase b, and also to the arrowhead (X_1) of the T_2 secondary phasor. The tail of the T_2 secondary phasor is connected to phase c, and also to the head of the T_3 secondary phasor.

The output-side line-to-neutral voltages can now be sketched, as in Fig. 6(e). These voltages have magnitude $110/\sqrt{3} = 63.5$ V . In comparison with the input line-to-neutral voltages, the output voltages lag by 30˚ and are reduced in magnitude by a factor of $2\sqrt{3} = 3.46$. In this circuit, the primary line-to-neutral voltages are reduced by the transformer turns ratio and then become the secondary line-to-line voltages. The wye-delta connection leads to a 30˚ phase lag.

PROBLEMS

1. A system containing a three-phase-delta input and a two-phase output with neutral

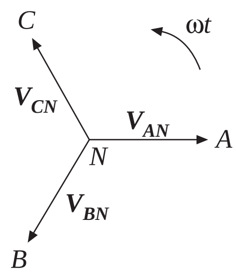
The transformer system shown below converts three-phase to two-phase with a neutral wire. The phasors for the primary-side voltages are as shown.

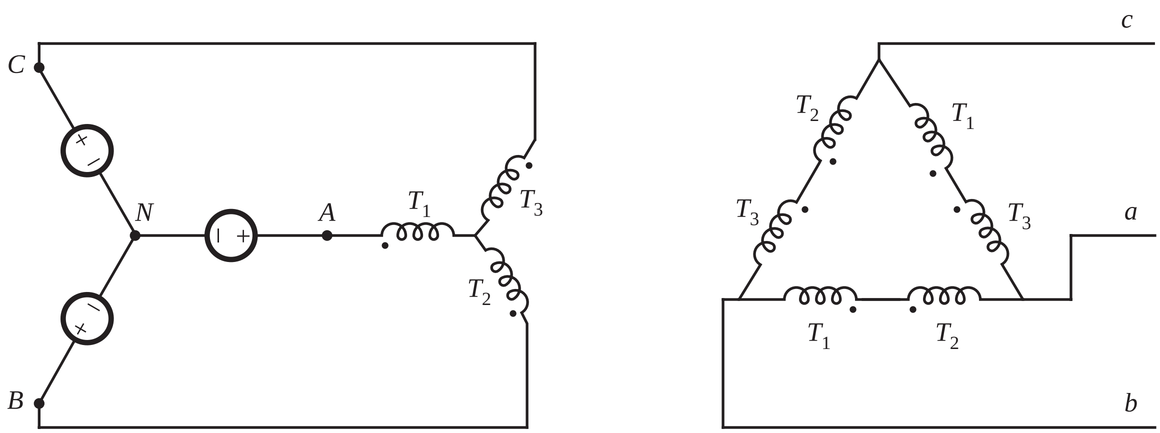


Carefully construct the phasor diagram for the secondary-side voltages, and hence determine the magnitudes and phases of the secondary voltages $V_{A'N'}$ and $V_{B'N'}$.

2. A wye – delta zigzag transformer connection

Each transformer has turns ratio 3:1:1. The phasors of the applied 3ø input voltages are shown.





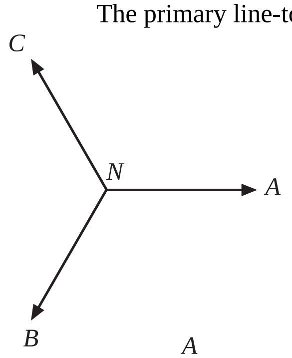
(a) Construct the phasors for the secondary voltages, and label each phasor with its transformer number (T_1, T_2, or T_3) and winding connections (X_1, X_2, etc.).

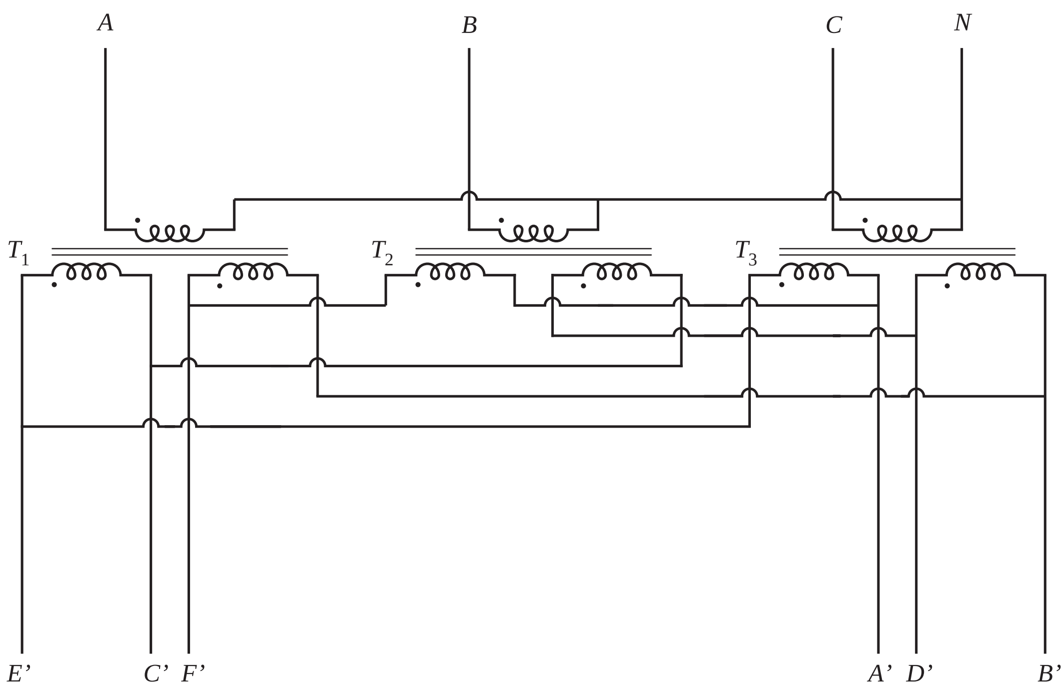
(b) Determine the magnitudes of the output line-line and line-neutral voltages.

(c) Determine the phase shift between the input and output line-neutral voltages.

3. A 3ø to 6ø transformer connection

Each transformer has turns ratio 2:1:1. The phasors of the applied 3ø input voltages are shown. The primary line-to-line voltage is 240V.





(a) Construct the phasors for the secondary voltages, and label each phasor with its transformer number (T_1, T_2, or T_3) and voltage magnitude. Label the six output phases.

(b) Determine the magnitudes of the output line-line voltages $V_{A'B'}$ and $V_{A'F'}$.

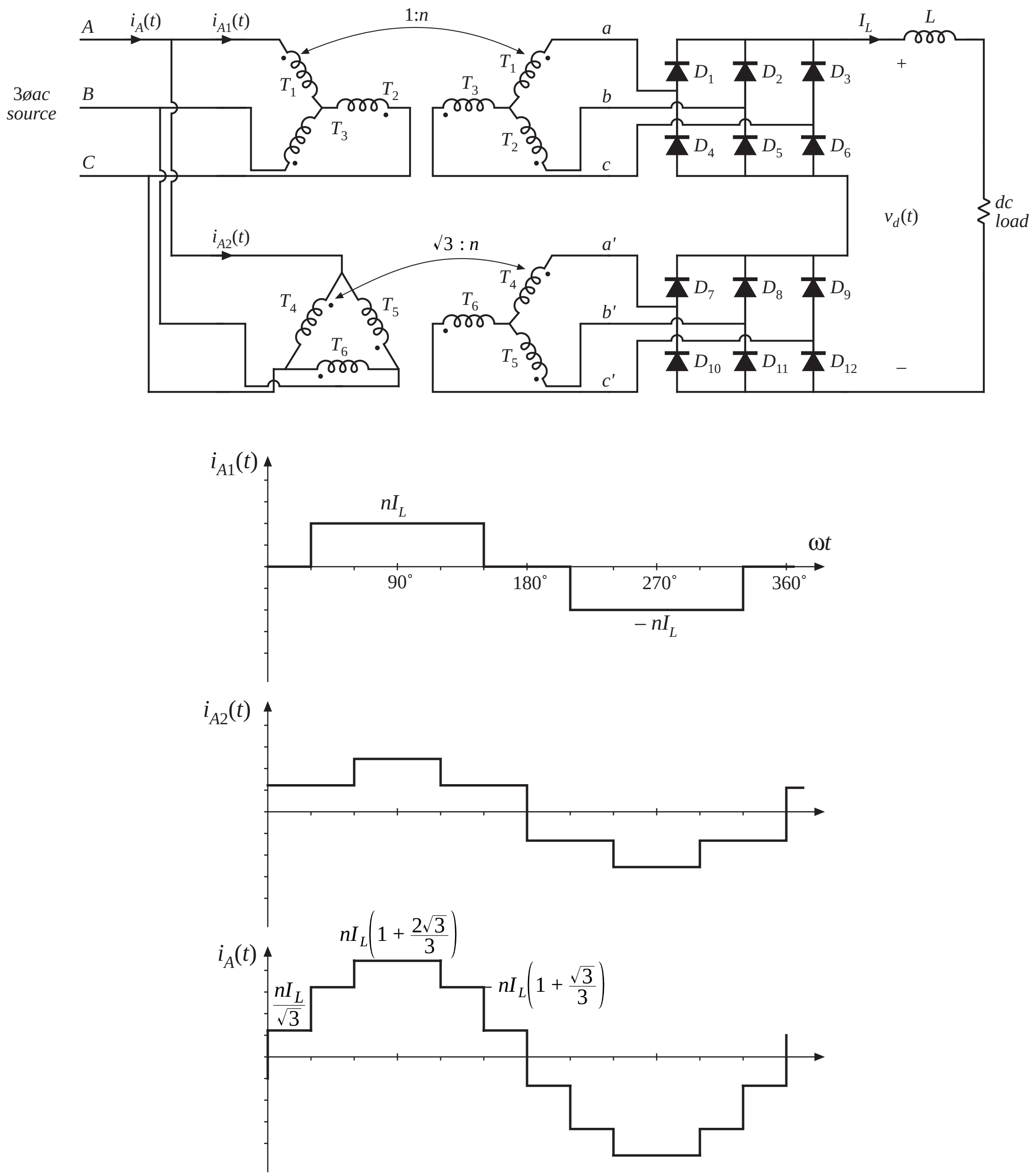
(c) Determine the phase shift between the input and output line-line voltages V_{AB} and $V_{A'B'}$.

4. A twelve-pulse rectifier

The circuit illustrated below is a twelve-pulse rectifier, containing a wye-wye transformer and a delta-wye transformer. Each transformer secondary is connected to a three-phase bridge rectifier. The output filter inductance L is very large, and hence its current is essentially purely dc with value I_L. Typical ac line current waveforms are also illustrated below.

(a) During every 30˚ interval, exactly four diode rectifiers are forward-biased by the applied voltages from the transformer secondaries. For example, there are six possible line-to-line voltage at the output of the T_1-T_2-T_3 secondary windings: v_{ab}, v_{bc}, v_{ca}, v_{ba}, v_{cb}, and v_{ac}. When v_{ab} is the most positive of these six voltages, then diodes D_1 and D_5 are forward-biased (turned on) while diodes D_2, D_3, D_4, and D_6 are reverse-biased (turned off). A similar mechanism governs conduction of diodes D_7 through D_{12}. Divide the 360˚ ac line period into twelve 30˚ intervals. For each interval, state which four diodes are forward-biased.

(b) Sketch the voltage waveform $v_d(t)$. Determine its maximum and minimum values, as functions of the applied 3øac source voltage V_{rms}.


A
B
C
3øac source
iA(t)
iA1(t)
iA2(t)
1:n
√3 : n
T1
T2
T3
T4
T5
T6
a
b
c
a′
b′
c′
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
D12
IL
L
+
–
vd(t)
dc load
iA1(t)
nIL
ωt
90˚
180˚
270˚
360˚
– nIL
iA2(t)
iA(t)
nIL/√3
nIL(1 + 2√3/3)
nIL(1 + √3/3)