Efficiency of a Transformer

The losses that take place in a transformer on load can be divided into two groups

(a) copper losses: These occur in the transformer primary and secondary windings. They may be calculated from

$$p_{copper} = p_{primary} + p_{secondary}$$

$$= I_1^2 R_1 + I_2^2 R_2$$

(b) Iron or core losses: These are constant losses due to hysteresis and eddy currents.

$$P_{iron} = cons\tan t$$

Thus the total power loss in a transformer is

$$p_{total} = p_{iron} + p_{copper}$$

$$= p_{iron} + I_1^2 R_1 + I_2^2 R_2$$

Since the efficiency of a transformer is the ration of the output power to the input power, it may be expressed as

$$\xi = \frac{output\ power}{input\ power} = \frac{output\ power}{output\ power\ + losses}$$

But $p = VI\cos\theta$ where $\cos\theta$ = power factor

Then $output\ power = V_2 I_2 \cos\theta$

where: $V_2 = voltage\ across\ the\ load$; $I_2 = current\ drawn\ by\ the\ load$.

Thus

$$\xi = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + p_{iron} + I_1^2 R_1 + I_2^2 R_2}$$

Efficiency of a Transformer on a Fraction Of The load

If current *I* passes through a resistance *R*, then the power dissipated in *R* is

$$p_{loss}(\,full-load\,)=I^2R$$

If *I* is reduced to half, then power dissipated in the same *R* is

$$p_{loss}(half-load\,)=\left(\frac{I}{2}\right)^2R$$

$$=\left(\frac{1}{2}\right)^2I^2R=\left(\frac{1}{2}\right)^2p_{loss}(\,full-load\,)$$

In general if *nth* amount of the current passes through the same R, then

$$p_{loss}(nth-load\,)=\left(\frac{1}{n}\right)^2p_{loss}(\,full-load\,)$$

and

$$p_{output}(nth-load\,)=n\,p_{output}(\,full-load\,)$$

Therefore the efficiency is

$$\xi=\frac{nV_2I_2\cos\theta}{nV_2I_2\cos\theta+p_{iron}+n^2I_1^2R_1+n^2I_2^2R_2}$$

Example

The primary and secondary windings of a 500 kVA transformer have resistances of $R_1=0.42\,\Omega$ and $R_2=0.0011\,\Omega$. The primary and secondary voltages are $V_1=6{,}600\ V$ and $V_2=400\ V$. The iron losses are $p_{iron}=2.9\ kW$. Calculate the efficiency of the transformer when it supplies (a) full load; (b) half load. Assume the load power factor to be *0.8* lagging.

Solution (a)

$$I_1(\,full - load\,) = \frac{kVA}{V_1} = \frac{500\,x1000}{6600} = 75.8\ A$$

$$I_2(\,full - load\,) = \frac{kVA}{V_2} = \frac{500\,x1000}{400} = 1{,}250\ A$$ Thus

$$p_{copper}(\,full - load\,) = I_1^2(\,fulll\ - load\,)R_1 + I_2^2(\,full\ - load\,)R_2$$

$$p_{copper} = (75.8)^2\,x0.42 + (1{,}250)^2\,x0.0011$$

or

$$p_{copper} = 2{,}415 + 1{,}720 = 4.135\ kW$$

Therefore the transformer total losses on full-load are

$$p_{total} = p_{iron} + p_{copper} = 2.9 + 4.135 = 7.035\ kW$$
$$output\ power(\,full - load\,) = V_2 I_2 \cos\theta = 500\,x0.8 = 400\ kW$$

Thus

$$\xi = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + p_{iron} + I_1^2 R_1 + I_2^2 R_2}$$

$$= \frac{400{,}000}{400{,}000 + 7{,}035} = 98.27\%$$

Solution (b)

When the transformer operates on half-load, the copper losses vary as follows

$$p_{copper}(\,half\ - load\,) = (0.5)^2\,p_{copper}(\,full\ - load\,)$$

$$= \left(0.5^2\right)4.135 = 1.034\ kW$$

The total power loss on half load is

$$p_{total}(\,half\ - load\,) = p_{iron} + p_{copper}(\,half\ - load\,)$$

$$= 2.9 + 1.034 = 3.934\ kW$$

The output power is now half of when the transformer was on full-load, i.e.

$$output\ power(half-load) = V_2 \frac{I_2}{2} \cos\theta = 250 x 0.8 = 200\ kW$$

$$\xi = \frac{0.5 V_2 I_2 \cos\theta}{0.5 V_2 I_2 \cos\theta + p_{iron} + (0.5)^2 I_1^2 R_1 + (0.5)^2 I_2^2 R_2}$$

$$= \frac{200}{200 + 3.934} = 98.07\%$$

Condition of Maximum Efficiency

Let the turn ratio of the transformer be

$$\sigma = \frac{N_2}{N_1}$$

Then

$$I_1 = \sigma I_2$$

and the efficiency is expressed as

$$\xi = \frac{V_2 I_2 xpf}{V_2 I_2 xpf + p_{iron} + \sigma^2 I_2^2 R_1 + I_2^2 R_2}$$

or

$$\xi = \frac{V_2 xpf}{V_2 xpf + \frac{P_{iron}}{I_2} + \sigma^2 I_2 R_1 + I_2 R_2}$$

For the efficiency to be maximum, the denominator must be minimum. In order to determine the load condition for which the denominator is minimum, we differentiate with respect to I_2 and equate to zero to obtain

$$\frac{d}{dI_2}\left(V_2xpf + \frac{p_{iron}}{I_2} + \sigma^2 I_2 R_1 + I_2 R_2\right) = 0$$

or

$$-\frac{p_{iron}}{I_2^2} + \sigma^2 R_1 + R_2 = 0$$

or

$$p_{iron} = \sigma^2 I_2^2 R_1 + I_2^2 R_2 = I_1^2 R_1 + I_2^2 R_2 = p_{copper}$$

Thus the condition for maximum efficiency is

$$p_{copper} = p_{iron}$$

Example
Find the output at which the efficiency of the transformer of the previous example is maximum and calculate its value.

Solution
Let *n* be the fraction of the load at which the efficiency is maximum, then

$$p_{copper}(nth-load) = n^2 p_{copper}(full-load) = n^2 x4.135\ kW$$

But at maximum efficiency

$$p_{copper} = p_{iron} = 2.9\ kW$$

Thus

$$2.9 = n^2 x4.135$$

or

$$n=0.837.$$

Therefore the output at maximum efficiency is

$$p_{output}(max\xi) = nxkVA = 0.837\ x500 = 418.5\ kVA$$

$$\xi = \frac{0.837\,x500\,x0.8}{0.837\,x500\,x0.8 + 2x2.9} = \frac{334.8}{334.8 + 5.8} = 98.3\%$$

Efficiency of a Transformer From Open- and Short-Circuit Tests

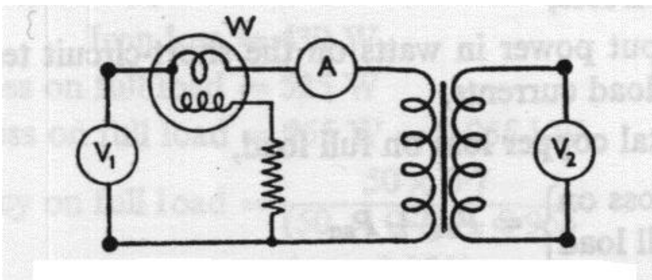
The open circuit test and short circuit tests enable the efficiency and voltage regulation to be determined without actually loading the transformer.

The power required to carry out these tests is small compared to the full-load output of the transformer.

The tests produce highly accurate results compared to load tests.

Open-Circuit Test

The transformer is connected as shown in Figure (1).



Open-circuit test on a transformer

$$V_1 = V_{rated} \equiv name\ plate$$

$$f = f_{rated} \equiv name\ plate$$

$$\frac{V_1(reading)}{V_2(reading)} = \frac{N_1}{N_2} = turns\ ratio$$

$A(reading) = I_1 = I_o$; usually $< 5\%$ $I_1(full-load)$.

Thus $p_{copper} = I^2(no-load)R < \dfrac{I^2(full-load)R}{400}$

Thus p_{copper} may be ignored.

$$W(reading) \cong p_{iron}$$

Short-Circuit Test
The transformer is connected as shown in Figure (2).

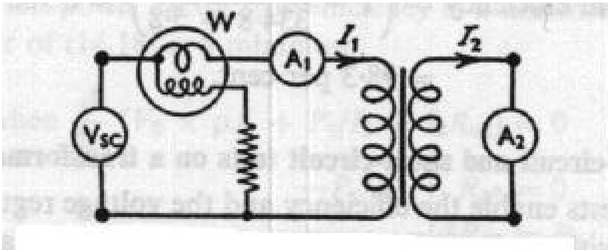


Figure (2): Short-circuit test on a transformer

$$V_1 = V_{sc} \equiv enough\ to\ ciculate\ full\ current$$

Usually $V_{sc} \cong \frac{1}{25} V_{rated}$

$$f = f_{rated} \equiv name\ plate$$

$$\frac{A_1(reading)}{A_2(reading)} = \frac{N_2}{N_1}$$

$$A_1(reading) = I_1(rated)$$

$$A_2(reading) = I_2(rated)$$

$$W(reading) = p_{copper} = I_1^2 R_1 + I_2^2 R_2$$

$$V_{sc} \cong \frac{1}{25} V_{rated}$$

$$p_{iron} << p_{copper}$$

Thus p_{iron} can be ignored.

p_{iron} are negligible because of the reduced applied voltage and therefore the flux.

Calculation of Efficiency From Open-Circuit and Short-Circuit Tests

Since

$$p_{iron} = p_{open-cicuit} = W_{open-circuit}$$

$$p_{copper} = p_{short-cicuit} = W_{short-circuit}$$

the efficiency on full-load is calculated from

$$\xi = \frac{full - load\ VA\ x\ pf}{full - load\ VA\ x\ pf + p_{open\text{-}circuit} + p_{short\text{-}circuit}}$$

And for an *nth*-load

$$\xi = \frac{n\ x\ full - load\ VA\ x\ pf}{n\ x\ full - load\ VA\ x\ pf + p_{open\text{-}circuit} + n^2\ x\ p_{short\text{-}circuit}}$$

Calculation of Voltage Regulation From Short-Circuit Tests

Consider the simplified equivalent circuit of a transformer shown in Figure (3).

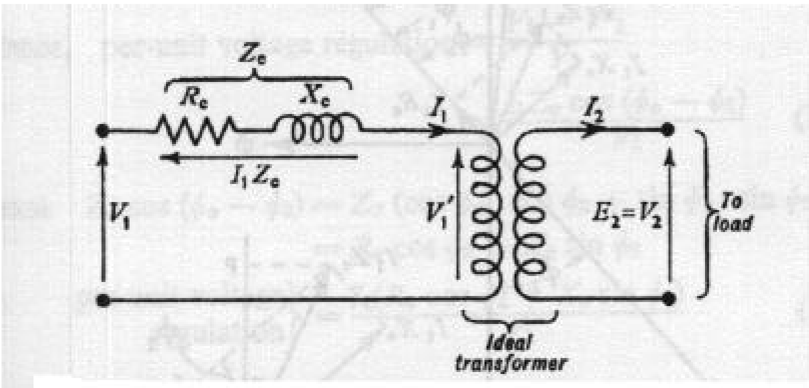


Figure (3): simplified equivalent circuit of a transformer

It was established that the voltage regulation of a transformer can be expressed as

$$V.R = \frac{I_1 Z_e \cos(\Phi_e - \Phi_2)}{V_1}$$

V_1 = primary rated voltage

I_1 = primary current

Z_e = equivalent impedance of the primary and secondary windings referred to the primary.

Φ_e= phase angle between I_1 and $I_1 Z_e$.

Φ_2= load phase angle.

From figure 3, it is clear that

$$p_{sc} = I_1 V_{sc} \cos\Phi_e$$

and

$$V_{sc} = I_1 Z_e;\ \text{since } V_2 = 0.$$

Thus

$$V.R = \frac{V_{sc} \cos(\Phi_e - \Phi_2)}{V_1}$$

Example

The following results were obtained from tests on a 50 kVA transformer

<u>Open-circuit test</u>
Primary voltage = 3300 V.
Secondary voltage = 400 V.
Primary power = 430 W.

<u>Short-circuit test</u>
Primary voltage = 124 V.
Primary current = 15.3 A.
Secondary current = full load value.
Primary power = 525 W.

Calculate
(a) the efficiencies at full load and half load for 0.7 power factor.

(b) the voltage regulation for 0.7 power factor, lagging and leading.

(c) the secondary terminal voltage for 0.7 power factor, lagging and leading.

Solution (a)

From the test results we have

$$p_{iron} = 430\ W$$
$$p_{copper}(\ full - load\ ) = 525\ W$$
$$p_{total}(\ full - load\ ) = 430 + 525 = 0.955\ kW$$

Thus

$$\xi(\ full\ - load\ ) = \frac{50x0.7}{50x0.7 + 0.955} = 97.34\%$$
$$\xi(\ half\ - load\ ) = \frac{0.5x50x0.7}{0.5x50x0.7 + 0.43 + 0.5^2 x525} = 96.9\%$$

Solution (b)

$$p_{sc} = I_1 V_{sc} \cos\Phi_e$$

or

$$\cos\Phi_e = \frac{p_{sc}}{I_1 V_{sc}} = \frac{525}{124x15.3} = 0.2765$$

or

$$\Phi_e = 74^o$$

Also from $\cos\Phi_2 = 0.7$ we find

$$\Phi_2 = 45.5^o$$

Thus

$$V.R = \frac{V_{sc}\cos(\ \Phi_e - \Phi_2\ )}{V_1}$$

for 0.7 pf lagging

$$VR = \frac{124\cos(\ 74 - 45.5\ )}{3300} = 3.3\%$$

for 0.7 pf leading

$$VR = \frac{124\cos(\ 74 + 45.5\ )}{3300} = -1.85\%$$

Solution (c)

Since the secondary voltage on open circuit (no-load) = rated voltage = 400 V, the secondary voltage on full-load is

$$V_2(\ full - load\ ) = 400(1 - VR)$$

for 0.7 power factor lagging

$$V_2(\ full - load\ ) = 400(1 - 0.033) = 386.8\ V$$

for 0.7 power factor leading

$$V_2(\ full - load\ ) = 400(1 + 0.0185) = 407.4\ V$$